



# TABLA DE DERIVADAS E INTEGRALES

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MIÉRCOLES  
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## NUMEROS NOTABLES

$$\pi = 3,14159...$$

$$\sqrt{2} = 1,414213...$$

$$e^{\pi} = 23,14069...$$

$$e^{\circ} = 15,15426...$$

$$e = 2,718281...$$

$$\sqrt{3} = 1,73205...$$

$$\pi^e = 22,45915...$$

$$\sqrt{e} = 1,64872...$$

$$\sqrt{\pi} = 1,77245... = \Gamma\left(\frac{1}{2}\right) \quad (\Gamma : \text{función gamma})$$

$$\Gamma\left(\frac{1}{3}\right) = 2,67893...$$

$$\Gamma\left(\frac{1}{4}\right) = 3,6250...$$

$$\gamma = 0,57721566... \quad (\text{constante de Euler})$$

$$1 \text{ radian} = \frac{180^\circ}{\pi} = 57,29577...^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ radianes} = 0,01745... \text{ radianes}$$

$$\text{números de Euler } (E_k) = \frac{\pi^{2k+1}}{2^{2k+2} (2k)!} \cdot E_k$$

$$\text{números de Bernoulli } (B_k) = \frac{\pi^{2k} \cdot (2^{2k} - 1)}{2 (2k)!} \cdot B_k$$

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## FUNCIONES TRIGONOMETRICAS CIRCULARES

### RELACIONES ENTRE FUNCIONES TRIGONOMETRICAS

$$1) \operatorname{sen}^2 x + \cos^2 x = 1$$

$$2) \operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}$$

$$3) \operatorname{cotg} x = \frac{\cos x}{\operatorname{sen} x}$$

$$4) \sec x = \frac{1}{\cos x}$$

$$5) \operatorname{cosec} x = \frac{1}{\operatorname{sen} x}$$

$$6) 1 + \operatorname{tg}^2 x = \sec^2 x$$

$$7) 1 + \operatorname{cotg}^2 x = \operatorname{cosec}^2 x$$

### FUNCIONES DE LA SUMA O DIFERENCIA DE ANGULOS

$$1) \operatorname{sen}(x \pm y) = \operatorname{sen} x \cdot \cos y \pm \operatorname{sen} y \cdot \cos x$$

$$2) \cos(x \pm y) = \cos x \cdot \cos y \mp \operatorname{sen} x \cdot \operatorname{sen} y$$

$$3) \operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \cdot \operatorname{tg} y}$$

### FUNCIONES DEL DUPLO DEL ANGULO

$$1) \operatorname{sen} 2x = 2 \cdot \operatorname{sen} x \cdot \cos x$$

$$2) \cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$$

$$3) \operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$

#### FUNCIONES DEL ANGULO MITAD

$$1) \sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}} \quad 2) \cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$3) \operatorname{tg}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 - \cos x} = \frac{1 - \cos x}{\sin x} = \operatorname{cosec} x - \cotg x$$

#### FUNCIONES POTENCIA

$$1) \sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

$$2) \cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

#### SUMA, DIFERENCIA Y PRODUCTO DE FUNCIONES

$$1) \sin x + \sin y = 2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\cos m\pi = (-1)^m$$

$$2) \sin x - \sin y = 2 \cdot \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$$

$$3) \cos x + \cos y = 2 \cdot \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$4) \cos x - \cos y = -2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$$

$$5) 2 \cdot \sin x \cdot \sin y = \cos(x-y) - \cos(x+y)$$

$$6) 2 \cdot \cos x \cdot \cos y = \cos(x-y) + \cos(x+y)$$

$$7) 2 \cdot \sin x \cdot \sin y = \sin(x-y) + \sin(x+y)$$

#### FUNCIONES TRIGONOMETRICAS HIPERBOLICAS

$$1) \sinh x = \frac{e^x - e^{-x}}{2}$$

$$2) \cosh x = \frac{e^x + e^{-x}}{2}$$

$$3) \operatorname{tgh} x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$4) \cotg h x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$5) \operatorname{sech} x = \frac{1}{\cosh x}$$

$$6) \operatorname{cosech} x = \frac{1}{\sinh x}$$

#### RELACIONES FUNDAMENTALES

$$1) \cosh^2 x - \sinh^2 x = 1$$

$$2) \operatorname{tgh}^2 x + \operatorname{sech}^2 x = 1$$

$$3) \cotg h^2 x - \operatorname{cosech}^2 x = 1$$

#### FUNCIONES DE LA SUMA Y DIFERENCIA DE ANGULOS

$$1) \sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$$

$$2) \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$3) \operatorname{tgh}(x \pm y) = \frac{\operatorname{tgh} x \pm \operatorname{tgh} y}{1 \pm \operatorname{tgh} x \operatorname{tgh} y}$$

#### FUNCIONES DEL DUPLO DEL ANGULO

$$1) \sinh 2x = 2 \cdot \sinh x \cdot \cosh x$$

$$2) \cosh 2x = \cosh^2 x + \sinh^2 x = 1 + 2 \sinh^2 x = 2 \cosh^2 x - 1$$

$$3) \operatorname{tgh} 2x = \frac{2 \operatorname{tgh} x}{1 - \operatorname{tgh}^2 x}$$

#### FUNCIONES DEL ANGULO MITAD

$$1) \sinh\left(\frac{x}{2}\right) = \pm \sqrt{\frac{\cosh x - 1}{2}} \quad \begin{cases} \text{si } x > 0, \text{ vale signo } + \\ \text{si } x < 0, \text{ vale signo } - \end{cases}$$

$$2) \cosh\left(\frac{x}{2}\right) = + \sqrt{\frac{1 + \cosh x}{2}}$$

$$3) \operatorname{tgh}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{\cosh x - 1}{\cosh x + 1}} = \frac{\sinh x}{1 + \cosh x} = \frac{\cosh x - 1}{\sinh x}$$

#### RELACION ENTRE LOS ARGUMENTOS HIPERBOLICOS Y LOGARITMICOS

$$1) \sinh^{-1} x = \ln\left(x + \sqrt{1 + x^2}\right); \quad \forall x$$

$$2) \operatorname{tgh}^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right); \quad |x| < 1$$

$$3) \operatorname{sech}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right); \quad 0 < x \leq 1$$

$$4) \cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right); \quad x \geq 1$$

$$5) \cotg h^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{x-1}\right); \quad |x| > 1$$

$$6) \operatorname{cosech}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right); \quad x \neq 0$$

# TABLA DE DERIVADAS

## REGLAS DE DERIVACION

\* Las funciones  $u$ ,  $v$  y  $w$  son derivables en  $x$ .

\*  $k$ ,  $r$ ,  $a$  y  $n$  son constantes reales

\*  $x$  es variable independiente

a) Regla de la cadena  $\frac{d}{dx} y = \frac{d}{du} y \cdot \frac{d}{dx} u$

b)  $\frac{d}{dx} y = \frac{1}{\frac{d}{dy} x}$

c)  $\frac{d}{dx} y = \frac{\frac{d}{du} y}{\frac{d}{du} x}$

FUNCION	DERIVADA
$k$	$0$
$x$	$1$
$kx$	$k$
$ku$	$k \frac{d}{dx} u$
$u^r$	$r u^{r-1} \frac{d}{dx} u$
$u + v - w$	$\frac{d}{dx} u + \frac{d}{dx} v - \frac{d}{dx} w$
$uv$	$\frac{d}{dx} u \cdot v + u \cdot \frac{d}{dx} v$
$uvw$	$\frac{d}{dx} u \cdot v \cdot w + u \cdot \frac{d}{dx} v \cdot w + u \cdot v \cdot \frac{d}{dx} w$
$\frac{u}{v}$	$\frac{\frac{d}{dx} u \cdot v - u \cdot \frac{d}{dx} v}{v^2}; \quad v \neq 0$
$\ln x$	$\frac{1}{x}$
$\ln u$	$\frac{1}{u} \cdot \frac{d}{dx} u$

FUNCION	DERIVADA
$\log_a u$	$\frac{1}{u \ln a} \cdot \frac{d}{dx} u \quad u > 0, a > 0, a \neq 1$
$e^x$	$e^x$
$e^u$	$e^u \cdot \frac{d}{dx} u$
$a^u$	$a^u \cdot \ln a \cdot \frac{d}{dx} u; \quad a > 0, a \neq 1$
$u^v$	$u^v \left( \frac{d}{dx} v \cdot \ln u + \frac{v}{u} \cdot \frac{d}{dx} u \right); \quad u > 0$
$\operatorname{sen} u$	$\cos u \cdot \frac{d}{dx} u$
$\cos u$	$-\operatorname{sen} u \cdot \frac{d}{dx} u$
$\operatorname{tg} u$	$\sec^2 u \cdot \frac{d}{dx} u$
$\operatorname{cotg} u$	$-\operatorname{cosec}^2 u \cdot \frac{d}{dx} u$
$\sec u$	$\sec u \cdot \operatorname{tg} u \cdot \frac{d}{dx} u$
$\operatorname{cosec} u$	$-\operatorname{cosec} u \cdot \operatorname{cotg} u \cdot \frac{d}{dx} u$
$\operatorname{senh} u$	$\cosh u \cdot \frac{d}{dx} u$
$\cosh u$	$\operatorname{senh} u \cdot \frac{d}{dx} u$
$\operatorname{tgh} u$	$\operatorname{sech}^2 u \cdot \frac{d}{dx} u$
$\operatorname{cotgh} u$	$-\operatorname{cosech}^2 u \cdot \frac{d}{dx} u$
$\operatorname{sech} u$	$-\operatorname{sech} u \cdot \operatorname{tgh} u \cdot \frac{d}{dx} u$
$\operatorname{cosech} u$	$-\operatorname{cosech} u \cdot \operatorname{cotgh} u \cdot \frac{d}{dx} u$

# TABLA DE INTEGRALES

## INTEGRALES INDEFINIDAS

### REGLAS PARA UNA INTEGRACION

\* Las  $f$ ,  $u$ ,  $v$  y  $w$  son funciones de  $x$ .

\*  $a$ ,  $b$ ,  $p$ ,  $q$ ,  $r$  y  $n$  son constantes,  $r$  es real y  $n$  es natural.

- 1  $\int a \, dx = ax$
- 2  $\int a f(x) \, dx = a \int f(x) \, dx$
- 3  $\int (u \pm v \pm w \pm \dots) \, dx = \int u \, dx \pm \int v \, dx \pm \int w \, dx \pm \dots$
- 4  $\int u \, dv = uv - \int v \, du$  Integración por partes
- 5  $\int f(ax) \, dx = \frac{1}{a} \int f(u) \, du$  Cambio de variable  $u = ax$
- 6  $\int F[f(x)] \, dx = \int F(u) \frac{dx}{du} \, du = \int \frac{F(u)}{f(x)} \, du$
- 7  $\int x^r \, dx = \frac{x^{r+1}}{r+1}$ ; Con  $r \neq -1$ . Para  $r = -1$  ver 8
- 8  $\int \frac{1}{x} \, dx = \ln |x| = \begin{cases} \ln x & \text{si } x > 0 \\ \ln(-x) & \text{si } x < 0 \end{cases}; x \neq 0$
- 9  $\int e^x \, dx = e^x$
- 10  $\int a^x \, dx = \frac{a^x}{\ln a} = a^x \log_a e$  Para  $a > 0$  y  $a \neq 1$
- 11  $\int \sin x \, dx = -\cos x$
- 12  $\int \cos x \, dx = \sin x$
- 13  $\int \operatorname{tg} x \, dx = \ln \sec x = -\ln \cos x$
- 14  $\int \operatorname{cotg} x \, dx = \ln \sin x$
- 15  $\int \sec x \, dx = \ln(\sec x + \operatorname{tg} x) = \ln \operatorname{tg} \left\{ \frac{x}{2} + \frac{\pi}{2} \right\}$
- 16  $\int \operatorname{cosec} x \, dx = \ln(\operatorname{cosec} x - \operatorname{cotg} x) = \ln \operatorname{tg} \frac{x}{2}$

FUNCION	DERIVADA
$\operatorname{sen}^{-1} u$ ( $\operatorname{arcsen} u$ )	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$\operatorname{cos}^{-1} u$ ( $\operatorname{arccos} u$ )	$-\frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$\operatorname{tg}^{-1} u$ ( $\operatorname{arctg} u$ )	$\frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$\operatorname{cotg}^{-1} u$ ( $\operatorname{arccotg} u$ )	$-\frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$\operatorname{sec}^{-1} u$ ( $\operatorname{arcsec} u$ )	$\frac{1}{u \cdot \sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$\operatorname{cosec}^{-1} u$ ( $\operatorname{arccosec} u$ )	$-\frac{1}{u \sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$\operatorname{senh}^{-1} u$ ( $\operatorname{arcsenh} u$ )	$\frac{1}{\sqrt{u^2+1}} \cdot \frac{d}{dx} u$
$\operatorname{cosh}^{-1} u$ ( $\operatorname{arccosh} u$ )	$\frac{1}{\sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$\operatorname{tgh}^{-1} u$ ( $\operatorname{arctgh} u$ )	$\frac{1}{1-u^2} \cdot \frac{d}{dx} u$
$\operatorname{cotgh}^{-1} u$ ( $\operatorname{arccotgh} u$ )	$\frac{1}{1-u^2} \cdot \frac{d}{dx} u$
$\operatorname{sech}^{-1} u$ ( $\operatorname{arcsech} u$ )	$-\frac{1}{u \sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$\operatorname{cosech}^{-1} u$ ( $\operatorname{arcosech}^{-1} u$ )	$-\frac{1}{u \sqrt{1+u^2}} \cdot \frac{d}{dx} u$

$$\begin{aligned}
17 \quad & \int \sec^2 x \, dx = \operatorname{tg} x \\
18 \quad & \int \operatorname{cosec}^2 x \, dx = -\operatorname{cotg} x \\
19 \quad & \int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x \\
20 \quad & \int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x \\
21 \quad & \int \operatorname{sen}^2 x \, dx = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} = \frac{1}{2} (x - \operatorname{sen} x \cos x) \\
22 \quad & \int \cos^2 x \, dx = \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} = \frac{1}{2} (x + \operatorname{sen} x \cos x) \\
23 \quad & \int \sec x \operatorname{tg} x \, dx = \sec x \\
24 \quad & \int \operatorname{cosec} x \operatorname{cotg} x \, dx = -\operatorname{cosec} x \\
25 \quad & \int \operatorname{senh} x \, dx = \cosh x \\
26 \quad & \int \cosh x \, dx = \operatorname{senh} x \\
27 \quad & \int \operatorname{tgh} x \, dx = \ln \cosh x \\
28 \quad & \int \operatorname{cotgh} x \, dx = \ln \operatorname{senh} x \\
29 \quad & \int \operatorname{sech} x \, dx = \operatorname{sen}^{-1} x (\operatorname{tgh} x) \quad \text{ó} \quad 2 \operatorname{tg}^{-1} e^x \\
30 \quad & \int \operatorname{cosech} x \, dx = \ln \operatorname{tgh} \frac{x}{2} \quad \text{ó} \quad -\operatorname{cotgh}^{-1} e^x \\
31 \quad & \int \operatorname{sech}^2 x \, dx = \operatorname{tgh} x \\
32 \quad & \int \operatorname{cosech}^2 x \, dx = -\operatorname{cotgh} x \\
33 \quad & \int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x \\
34 \quad & \int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x \\
35 \quad & \int \operatorname{senh}^2 x \, dx = \frac{\operatorname{senh} 2x}{4} - \frac{x}{2} = \frac{1}{2} (\operatorname{senh} x \cosh x - x) \\
36 \quad & \int \cosh^2 x \, dx = \frac{\operatorname{senh} 2x}{4} + \frac{x}{2} = \frac{1}{2} (\operatorname{senh} x \cosh x + x) \\
37 \quad & \int \operatorname{sech} x \operatorname{tgh} x \, dx = -\operatorname{sech} x \\
38 \quad & \int \operatorname{cosech} x \operatorname{cotgh} x \, dx = -\operatorname{cosech} x
\end{aligned}$$

$$\begin{aligned}
39 \quad & \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{tg}^{-1} \frac{x}{a} \\
40 \quad & \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left( \frac{x-a}{x+a} \right) = -\frac{1}{a} \operatorname{cotgh}^{-1} \frac{x}{a} \quad x^2 > a^2 \\
41 \quad & \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left( \frac{x+a}{a-x} \right) = \frac{1}{a} \operatorname{tgh}^{-1} \frac{x}{a} \quad x^2 < a^2 \\
42 \quad & \int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{sen}^{-1} \frac{x}{a} \\
43 \quad & \int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left( x + \sqrt{a^2 + x^2} \right) \quad \text{ó} \quad \operatorname{senhr}^{-1} \frac{x}{a} \\
44 \quad & \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left( x + \sqrt{x^2 - a^2} \right) \quad \text{ó} \quad \operatorname{cosh}^{-1} \frac{x}{a} \\
45 \quad & \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{sec}^{-1} \left| \frac{x}{a} \right| \\
46 \quad & \int \frac{dx}{x \sqrt{x^2 + a^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \\
47 \quad & \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) \\
48 \quad & \int f^{(n)} g \, dx = f^{(n-1)} g - f^{(n-2)} g' + f^{(n-3)} g'' - \dots + (-1)^n \int f g^{(n)} \, dx
\end{aligned}$$

# METODO DE SUSTITUCION

$$\begin{aligned}
49 \quad & \int F(ax + b) \, dx = \frac{1}{a} \int F(u) \, du & u = ax + b \\
50 \quad & \int F(\sqrt{ax + b}) \, dx = \frac{2}{a} \int u F(u) \, du & u = \sqrt{ax + b} \\
51 \quad & \int F(\sqrt[n]{ax + b}) \, dx = \frac{n}{a} \int u^{n-1} F(u) \, du & u = \sqrt[n]{ax + b} \\
52 \quad & \int F(\sqrt{a^2 - x^2}) \, dx = a \int F(a \cos u) \cos u \, du & x = a \operatorname{sen} u \\
53 \quad & \int F(\sqrt{x^2 + a^2}) \, dx = a \int F(a \sec u) \sec^2 u \, du & x = a \operatorname{tg} u \\
54 \quad & \int F(\sqrt{x^2 - a^2}) \, dx = a \int F(a \sec u) \sec u \operatorname{tg} u \, du & x = a \sec u \\
55 \quad & \int F(e^{ax}) \, dx = \frac{1}{a} \int \frac{F(u)}{u} \, du & u = e^{ax} \\
56 \quad & \int F(\ln u) \, dx = \int F(u) e^u \, du & u = \ln u
\end{aligned}$$

$$57 \int F\left(\sin^{-1} \frac{x}{a}\right) dx = a \int F(u) \cos u \, du \quad u = \sin^{-1} \frac{x}{a}$$

Para otras funciones trigonométricas recíprocas se obtienen similares resultados.

$$58 \int F(\sin x, \cos x) dx = 2 \int F\left(\frac{2u}{1+u^2}, \frac{1-u^2}{1+u^2}\right) \frac{du}{1+u^2} \quad u = \tan \frac{x}{2}$$

### INTEGRALES INDEFINIDAS CLASIFICADAS POR LA FORMA

#### INTEGRALES CON $ax + b$

$$59 \int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b)$$

$$60 \int \frac{x \, dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2} \ln(ax+b)$$

$$61 \int \frac{x^2 \, dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3} \ln(ax+b)$$

$$62 \int \frac{x^3 \, dx}{ax+b} = \frac{(ax+b)^3}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4} \ln(ax+b)$$

$$63 \int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln\left(\frac{x}{ax+b}\right)$$

$$64 \int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln\left(\frac{ax+b}{x}\right)$$

$$65 \int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3} \ln\left(\frac{x}{ax+b}\right)$$

$$66 \int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

$$67 \int \frac{x \, dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln(ax+b)$$

$$68 \int \frac{x^2 \, dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3} \ln(ax+b)$$

$$69 \int \frac{x^3 \, dx}{(ax+b)^2} = \frac{(ax+b)^2}{2a^4} - \frac{3b(ax+b)}{a^4} + \frac{b^2}{a^4(ax+b)} + \frac{3b^2}{a^4} \ln(ax+b)$$

$$70 \int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2} \ln\left(\frac{x}{ax+b}\right)$$

$$71 \int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3} \ln\left(\frac{ax+b}{x}\right)$$

$$72 \int \frac{dx}{x^3(ax+b)^2} = -\frac{(ax+b)^2}{2b^4x^2} - \frac{a^3x}{b^4(ax+b)} + \frac{3a(ax+b)}{b^4x} - \frac{3a^2}{b^4} \ln\left(\frac{ax+b}{x}\right)$$

$$73 \int \frac{dx}{(ax+b)^3} = \frac{-1}{2(ax+b)^2}$$

$$74 \int \frac{x \, dx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^2(ax+b)^2}$$

$$75 \int \frac{x^2 \, dx}{(ax+b)^3} = \frac{2b}{a^3(ax+b)} - \frac{b^2}{2a^3(ax+b)^2} + \frac{1}{a^3} \ln(ax+b)$$

$$76 \int \frac{x^3 \, dx}{(ax+b)^3} = \frac{x}{a^3} - \frac{3b^2}{a^4(ax+b)} + \frac{b^3}{2a^4(ax+b)^2} - \frac{3b}{a^4} \ln(ax+b)$$

$$77 \int \frac{dx}{x(ax+b)^3} = \frac{a^2x^2}{2b^3(ax+b)^2} - \frac{2ax}{b^3(ax+b)} - \frac{1}{b^3} \ln\left(\frac{ax+b}{x}\right)$$

$$78 \int \frac{dx}{x^2(ax+b)^3} = \frac{-a}{2b^2(ax+b)^2} - \frac{2a}{b^3(ax+b)} - \frac{1}{b^3x} + \frac{3a}{b^4} \ln\left(\frac{ax+b}{x}\right)$$

$$79 \int \frac{dx}{x^3(ax+b)^3} = \frac{a^4x^2}{2b^5(ax+b)^2} - \frac{4a^3x}{b^5(ax+b)} - \frac{(ax+b)^2}{2b^5x^2} - \frac{6a^2}{b^5} \ln\left(\frac{ax+b}{x}\right)$$

$$80 \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a}; \quad \text{Si } n = -1 \text{ véase 59.}$$

$$81 \int x(ax+b)^n dx = \frac{(ax+b)^{n+2}}{(n+2)a^2} - \frac{b(ax+b)^{n+1}}{(n+1)a^2}; \quad n \neq -1, -2. \text{ Si } n = -1 \text{ ó } -2 \text{ véase 62 ó}$$

67, respectivamente.

$$82 \int x^2(ax+b)^n dx = \frac{(ax+b)^{n+3}}{(n+3)a^3} - \frac{2b(ax+b)^{n+2}}{(n+2)a^3} + \frac{b^2(ax+b)^{n+1}}{(n+1)a^3}, \quad n \neq -1, -2, -3.$$

Si  $n = -1, -2$  ó  $-3$  véase 61, 68 ó 75, respectivamente.

$$83 \int x^m(ax+b)^n dx = \begin{cases} \frac{x^{m+1}(ax+b)^n}{m+n+1} + \frac{nb}{m+n+1} \int x^m(ax+b)^{n-1} dx \\ \frac{x^m(ax+b)^{n+1}}{(m+n+1)a} - \frac{mb}{(m+n+1)a} \int x^{m-1}(ax+b)^n dx \\ \frac{-x^{m+1}(ax+b)^{n+1}}{(n+1)b} + \frac{m+n+2}{(n+1)b} \int x^m(ax+b)^{n+1} dx \end{cases}$$

#### INTEGRALES CON $\sqrt{ax+b}$

$$84 \int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$$

$$85 \int \frac{x \, dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2} \sqrt{ax+b}$$

$$86 \int \frac{x^2 \, dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2 - 4abx + 8b^2)}{15a^2} \sqrt{ax+b}$$

$$87 \int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{b} \ln\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\ \frac{2}{\sqrt{-b}} \operatorname{tg}^{-1} \sqrt{\frac{ax+b}{-b}} \end{cases} \quad b \neq 0$$

$$88 \int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax+b}} \quad \text{Véase 87 } b \neq 0$$



$$89 \int \sqrt{ax+b} dx = \frac{2\sqrt{(ax+b)^3}}{3a}$$

$$90 \int x \sqrt{ax+b} dx = \frac{2(3ax-2b)}{15a^2} \sqrt{(ax+b)^3}$$

$$91 \int x^2 \sqrt{ax+b} dx = \frac{2(15a^2x^2 - 12abx + 8b^2)}{105a^3} \sqrt{(ax+b)^3}$$

$$92 \int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}} \quad \text{Véase 87}$$

$$93 \int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}} \quad \text{Véase 87}$$

$$94 \int \frac{x^m dx}{\sqrt{ax+b}} = \frac{2x^m \sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a} \int \frac{x^{m-1} dx}{\sqrt{ax+b}}$$

$$95 \int \frac{dx}{x^m \sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)b x^{m-1}} - \frac{(2m-3)a}{(2m-2)b} \int \frac{dx}{x^{m-1} \sqrt{ax+b}} \quad m \neq 1$$

$$96 \int x^m \sqrt{ax+b} dx = \frac{2x^m}{(2m+3)a} \sqrt{(ax+b)^3} - \frac{2mb}{(2m+3)a} \int x^{m-1} \sqrt{ax+b} dx$$

$$97 \int \frac{\sqrt{ax+b}}{x^m} dx = -\frac{\sqrt{ax+b}}{(m-1)x^{m-1}} + \frac{a}{2(m-1)} \int \frac{dx}{x^{m-1} \sqrt{ax+b}}$$

$$98 \int \frac{\sqrt{ax+b}}{x^m} dx = -\frac{\sqrt{(ax+b)^3}}{(m-1)b x^{m-1}} - \frac{(2m-5)a}{(2m-2)b} \int \frac{\sqrt{ax+b}}{x^{m-1}} dx$$

$$99 \int \sqrt{(ax+b)^m} dx = \frac{2\sqrt{(ax+b)^{m+2}}}{a(m+2)}$$

$$100 \int x \sqrt{(ax+b)^m} dx = \frac{2\sqrt{(ax+b)^{m+4}}}{a^2(m+4)} - \frac{2b\sqrt{(ax+b)^{m+2}}}{a^2(m+2)}$$

$$101 \int x^2 \sqrt{(ax+b)^m} dx = \frac{2\sqrt{(ax+b)^{m+6}}}{a^3(m+6)} - \frac{4b\sqrt{(ax+b)^{m+4}}}{a^3(m+4)} + \frac{2b^2\sqrt{(ax+b)^{m+2}}}{a^3(m+2)}$$

$$102 \int \frac{\sqrt{(ax+b)^m}}{x} dx = 2\frac{\sqrt{(ax+b)^m}}{m} + b \int \frac{\sqrt{(ax+b)^{m-2}}}{x} dx$$

$$103 \int \frac{\sqrt{(ax+b)^m}}{x^2} dx = -\frac{\sqrt{(ax+b)^{m+2}}}{bx} + \frac{ma}{2b} \int \frac{\sqrt{(ax+b)^m}}{x} dx$$

$$104 \int \frac{dx}{x \sqrt{(ax+b)^m}} = \frac{2}{(m-2)b \sqrt{(ax+b)^{m-2}}} + \frac{1}{b} \int \frac{dx}{x^m \sqrt{(ax+b)^{m-2}}}$$

# INTEGRALES CON $ax+b$ y $px+q$ , donde $bp-aq \neq 0$

$$105 \int \frac{1}{(ax+b)(px+q)} dx = \frac{1}{bp-aq} \ln \left( \frac{px+q}{ax+b} \right)$$

$$106 \int \frac{x}{(ax+b)(px+q)} dx = \frac{1}{bp-aq} \left\{ \frac{b}{a} \ln(ax+b) - \frac{q}{p} \ln(px+q) \right\}$$

$$107 \int \frac{1}{(ax+b)^2(px+q)} dx = \frac{1}{bp-aq} \left\{ \frac{1}{ax+b} + \frac{p}{bp-aq} \ln \left( \frac{px+q}{ax+b} \right) \right\}$$

$$108 \int \frac{x}{(ax+b)^2(px+q)} dx = \frac{1}{bp-aq} \left\{ \frac{q}{bp-aq} \ln \left( \frac{ax+b}{px+q} \right) - \frac{b}{a(ax+b)} \right\}$$

$$109 \int \frac{x^2 dx}{(ax+b)^2(px+q)} = \frac{b^2}{(bp-aq)a^2(ax+b)} + \frac{1}{(bp-aq)^2} \left\{ \frac{q^2}{p} \ln(px+q) + \frac{b(bp-aq)}{a^2} \ln(ax+b) \right\}$$

$$110 \int \frac{dx}{(ax+b)^m(px+q)^n} = \frac{-1}{(n-1)(bp-aq)} \left\{ \frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2) \int \frac{dx}{(ax+b)^m(px+q)^{n-1}} \right\}$$

$$111 \int \frac{ax+b}{px+q} dx = \frac{ax}{p} + \frac{bp-aq}{p^2} \ln(px+q)$$

$$112 \int \frac{(ax+b)^m}{(px+q)^n} dx = \begin{cases} \frac{-1}{(n-1)(bp-aq)} \left\{ \frac{(ax+b)^{m+1}}{(px+q)^{n-1}} + a(n-m-2) \int \frac{(ax+b)^m}{(px+q)^{n-1}} dx \right\} \\ \frac{-1}{(n-m-1)p} \left\{ \frac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq) \int \frac{(ax+b)^{m-1}}{(px+q)^{n-1}} dx \right\} \\ \frac{-1}{(n-1)p} \left\{ \frac{(ax+b)^m}{(px+q)^{n-1}} - m \int \frac{(ax+b)^{m-1}}{(px+q)^{n-1}} dx \right\} \end{cases}$$

$m, n \in \mathbb{N}$

# INTEGRALES CON $\sqrt{ax+b}$ y $px+q$

$$113 \int \frac{px+q}{\sqrt{ax+b}} dx = \frac{2(apx+3aq-2bp)}{3a^2} \sqrt{ax+b}$$

$$114 \int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{bp-aq}\sqrt{p}} \ln \left( \frac{\sqrt{p(ax+b)} - \sqrt{bp-aq}}{\sqrt{p(ax+b)} + \sqrt{bp-aq}} \right) \\ \frac{2}{\sqrt{aq-bp}\sqrt{p}} \operatorname{tg}^{-1} \sqrt{\frac{p(ax+b)}{aq-bp}} \end{cases}$$

$$115 \int \frac{\sqrt{ax+b}}{px+q} dx = \begin{cases} \frac{2\sqrt{ax+b}}{p} + \frac{\sqrt{bp-aq}}{p\sqrt{p}} \ln \left( \frac{\sqrt{p(ax+b)} - \sqrt{bp-aq}}{\sqrt{p(ax+b)} + \sqrt{bp-aq}} \right) \\ \frac{2\sqrt{ax+b}}{p} - \frac{2\sqrt{aq-bp}}{p\sqrt{p}} \operatorname{tg}^{-1} \sqrt{\frac{p(ax+b)}{aq-bp}} \end{cases}$$

$$116 \int (px+q)^n \sqrt{ax+b} dx = \frac{2(px+q)^{n+1} \sqrt{ax+b}}{(2n+3)p} + \frac{bp-aq}{(2n+3)p} \int \frac{(px+q)^n}{\sqrt{ax+b}} dx$$

$$117 \int \frac{dx}{(px+q)^n \sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)Xaq-bpX(px+q)^{n-1}} + \frac{(2n-3)a}{2(n-1)Xaq-bp} \int \frac{dx}{(px+q)^{n-1} \sqrt{ax+b}}$$

$$118 \int \frac{(px+q)^n}{\sqrt{ax+b}} dx = \frac{2(px+q)^n \sqrt{ax+b}}{(2n+1)a} + \frac{2n(aq-bp)}{(2n+1)a} \int \frac{(px+q)^{n-1}}{\sqrt{ax+b}} dx$$

$$119 \int \frac{\sqrt{ax+b}}{(px+q)^n} dx = \frac{-\sqrt{ax+b}}{(n-1)p(px+q)^{n-1}} + \frac{a}{2(n-1)p} \int \frac{dx}{(px+q)^{n-1} \sqrt{ax+b}}$$

#### INTEGRALES CON $\sqrt{ax+b}$ y $\sqrt{px+q}$

$$120 \int \frac{dx}{\sqrt{(ax+b)(px+q)}} = \begin{cases} \frac{2}{\sqrt{ap}} \ln \left( \sqrt{a(px+q)} + \sqrt{p(ax+b)} \right) \\ \frac{2}{\sqrt{-ap}} \operatorname{tg}^{-1} \sqrt{\frac{-p(ax+b)}{a(px+q)}} \end{cases}$$

$$121 \int \frac{x dx}{\sqrt{(ax+b)(px+q)}} = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap} \int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$122 \int \sqrt{(ax+b)(px+q)} dx = \frac{2apx+bp+aq}{4ap} \sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap} \int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$123 \int \frac{px+q}{\sqrt{ax+b}} dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{(aq-bp)}{2a} \int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$124 \int \frac{dx}{(px+q) \sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$$

#### INTEGRALES CON $x^2 + a^2$

$$125 \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{tg}^{-1} \frac{x}{a}$$

$$126 \int \frac{x dx}{x^2 + a^2} = \frac{1}{2} \ln(x^2 + a^2)$$

$$127 \int \frac{x^2 dx}{x^2 + a^2} = x - a \operatorname{tg}^{-1} \frac{x}{a}$$

$$128 \int \frac{x^3 dx}{x^2 + a^2} = \frac{x^2}{2} - \frac{a^2}{2} \ln(x^2 + a^2)$$

$$129 \int \frac{dx}{x(x^2 + a^2)} = \frac{1}{2a^2} \ln \left( \frac{x^2}{x^2 + a^2} \right)$$

$$130 \int \frac{dx}{x^2(x^2 + a^2)} = -\frac{1}{a^2 x} - \frac{1}{a^4} \operatorname{tg}^{-1} \frac{x}{a}$$

$$131 \int \frac{dx}{x^3(x^2 + a^2)} = -\frac{1}{2a^2 x^2} - \frac{1}{2a^4} \ln \left( \frac{x^2}{x^2 + a^2} \right)$$

$$132 \int \frac{dx}{(x^2 + a^2)^2} = \frac{x}{2a^2(x^2 + a^2)} + \frac{1}{2a^3} \operatorname{tg}^{-1} \frac{x}{a}$$

$$133 \int \frac{x dx}{(x^2 + a^2)^2} = \frac{-1}{2(x^2 + a^2)}$$

$$134 \int \frac{x^2 dx}{(x^2 + a^2)^2} = \frac{-x}{2(x^2 + a^2)} + \frac{1}{2a} \operatorname{tg}^{-1} \frac{x}{a}$$

$$135 \int \frac{x^3 dx}{(x^2 + a^2)^2} = \frac{a^2}{2(x^2 + a^2)} + \frac{1}{2} \ln(x^2 + a^2)$$

$$136 \int \frac{dx}{x(x^2 + a^2)^2} = \frac{1}{2a^2(x^2 + a^2)} + \frac{1}{2a^4} \ln \left( \frac{x^2}{x^2 + a^2} \right)$$

$$137 \int \frac{dx}{x^2(x^2 + a^2)^2} = -\frac{1}{a^4 x} - \frac{x}{2a^4(x^2 + a^2)} - \frac{3}{2a^6} \operatorname{tg}^{-1} \frac{x}{a}$$

$$138 \int \frac{dx}{x^3(x^2 + a^2)^2} = -\frac{1}{2a^4 x^2} - \frac{1}{2a^4(x^2 + a^2)} - \frac{1}{a^6} \ln \left( \frac{x^2}{x^2 + a^2} \right)$$

$$139 \int \frac{dx}{(x^2 + a^2)^n} = \frac{x}{2(n-1)a^2(x^2 + a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(x^2 + a^2)^{n-1}}; \text{ si } n = 1 \text{ ver } 125$$

$$140 \int \frac{x dx}{(x^2 + a^2)^n} = \frac{-1}{2(n-1)(x^2 + a^2)^{n-1}}; \text{ si } n = 1 \text{ ver } 126$$

$$141 \int \frac{dx}{x(x^2 + a^2)^n} = \frac{1}{2(n-1)a^2(x^2 + a^2)^{n-1}} + \frac{1}{a^2} \int \frac{dx}{x(x^2 + a^2)^{n-1}}; \text{ si } n = 1 \text{ ver } 129$$

$$142 \int \frac{x^m dx}{(x^2 + a^2)^n} = \int \frac{x^{m-2} dx}{(x^2 + a^2)^{n-1}} - a^2 \int \frac{x^{m-2} dx}{(x^2 + a^2)^n}$$

$$143 \int \frac{dx}{x^m(x^2 + a^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^m(x^2 + a^2)^{n-1}} - \frac{1}{a^2} \int \frac{dx}{x^{m-2}(x^2 + a^2)^n}$$

#### INTEGRALES CON $x^2 - a^2$ , $x^2 > a^2$

$$144 \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left( \frac{x-a}{x+a} \right) \quad \text{o} \quad -\frac{1}{a} \operatorname{cotgh}^{-1} \frac{x}{a}$$

$$145 \int \frac{x dx}{x^2 - a^2} = \frac{1}{2} \ln(x^2 - a^2)$$

$$146 \int \frac{x^2 dx}{x^2 - a^2} = x + \frac{a^2}{2} \ln \left( \frac{x-a}{x+a} \right)$$

$$147 \int \frac{x^3 dx}{x^2 - a^2} = \frac{x^2}{2} + \frac{a^2}{2} \ln(x^2 - a^2)$$

$$148 \int \frac{dx}{x(x^2 - a^2)} = \frac{1}{2a^2} \ln \left( \frac{x^2 - a^2}{x^2} \right)$$

$$149 \int \frac{dx}{x^2(x^2 - a^2)} = \frac{1}{a^2 x} + \frac{1}{2a^4} \ln \left( \frac{x-a}{x+a} \right)$$

$$150 \int \frac{dx}{x^3(x^2 - a^2)} = \frac{1}{2a^2 x^2} - \frac{1}{2a^4} \ln \left( \frac{x^2}{x^2 - a^2} \right)$$

$$151 \int \frac{dx}{(x^2 - a^2)^2} = \frac{-x}{2a^2(x^2 - a^2)} - \frac{1}{4a^3} \ln \left( \frac{x-a}{x+a} \right)$$

$$\begin{aligned}
152 \quad & \int \frac{x \, dx}{(x^2 - a^2)^2} = \frac{-1}{2(x^2 - a^2)} \\
153 \quad & \int \frac{x^2 \, dx}{(x^2 - a^2)^2} = \frac{-x}{2(x^2 - a^2)} + \frac{1}{4a} \ln \left( \frac{x-a}{x+a} \right) \\
154 \quad & \int \frac{x^3 \, dx}{(x^2 - a^2)^2} = \frac{-a^2}{2(x^2 - a^2)} + \frac{1}{2} \ln(x^2 - a^2) \\
155 \quad & \int \frac{dx}{x(x^2 - a^2)^2} = \frac{-1}{2a^2(x^2 - a^2)} + \frac{1}{2a^4} \ln \left( \frac{x^2}{x^2 - a^2} \right) \\
156 \quad & \int \frac{dx}{x^2(x^2 - a^2)^2} = -\frac{1}{a^4 x} - \frac{x}{2a^4(x^2 - a^2)} - \frac{3}{4a^6} \ln \left( \frac{x-a}{x+a} \right) \\
157 \quad & \int \frac{dx}{x^3(x^2 - a^2)^2} = -\frac{1}{2a^4 x^2} - \frac{1}{2a^4(x^2 - a^2)} + \frac{1}{a^6} \ln \left( \frac{x^2}{x^2 - a^2} \right) \\
158 \quad & \int \frac{dx}{(x^2 - a^2)^n} = \frac{-x}{2(n-1)(x^2 - a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(x^2 - a^2)^{n-1}} \\
159 \quad & \int \frac{x \, dx}{(x^2 - a^2)^n} = \frac{-1}{2(n-1)(x^2 - a^2)^{n-1}} \\
160 \quad & \int \frac{dx}{x(x^2 - a^2)^n} = \frac{-1}{2(n-1)a^2(x^2 - a^2)^{n-1}} - \frac{1}{a^2} \int \frac{dx}{x(x^2 - a^2)^{n-1}} \\
161 \quad & \int \frac{x^n \, dx}{(x^2 - a^2)^n} = \int \frac{x^{n-2} \, dx}{(x^2 - a^2)^{n-1}} + a^2 \int \frac{x^{n-2} \, dx}{(x^2 - a^2)^n} \\
162 \quad & \int \frac{dx}{x^n(x^2 - a^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^{n-2}(x^2 - a^2)^n} - \frac{1}{a^2} \int \frac{dx}{x^n(x^2 - a^2)^{n-1}}
\end{aligned}$$

#### INTEGRALES CON $a^2 - x^2, x^2 < a^2$

$$\begin{aligned}
163 \quad & \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left( \frac{a+x}{a-x} \right) \quad \text{ó} \quad \frac{1}{a} \operatorname{tgtr}^{-1} \frac{x}{a} \\
164 \quad & \int \frac{x \, dx}{a^2 - x^2} = -\frac{1}{2} \ln(a^2 - x^2) \\
165 \quad & \int \frac{x^2 \, dx}{a^2 - x^2} = -x + \frac{a}{2} \ln \left( \frac{a+x}{a-x} \right) \\
166 \quad & \int \frac{x^3 \, dx}{a^2 - x^2} = -\frac{x^2}{2} - \frac{a^2}{2} \ln(a^2 - x^2) \\
167 \quad & \int \frac{dx}{x(a^2 - x^2)} = \frac{1}{2a^2} \ln \left( \frac{x^2}{a^2 - x^2} \right) \\
168 \quad & \int \frac{dx}{x^2(a^2 - x^2)} = -\frac{1}{a^2 x} + \frac{1}{2a^3} \ln \left( \frac{a+x}{a-x} \right) \\
169 \quad & \int \frac{dx}{x^3(a^2 - x^2)} = -\frac{1}{2a^2 x^2} + \frac{1}{2a^4} \ln \left( \frac{x^2}{a^2 - x^2} \right) \\
170 \quad & \int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln \left( \frac{a+x}{a-x} \right) \\
171 \quad & \int \frac{x \, dx}{(a^2 - x^2)^2} = \frac{1}{2(a^2 - x^2)}
\end{aligned}$$

$$\begin{aligned}
172 \quad & \int \frac{x^2 \, dx}{(a^2 - x^2)^2} = \frac{x}{2(a^2 - x^2)} - \frac{1}{4a} \ln \left( \frac{a+x}{a-x} \right) \\
173 \quad & \int \frac{x^3 \, dx}{(a^2 - x^2)^2} = \frac{a^2}{2(a^2 - x^2)} + \frac{1}{2} \ln(a^2 - x^2) \\
174 \quad & \int \frac{dx}{x(a^2 - x^2)^2} = \frac{1}{2a^2(a^2 - x^2)} + \frac{1}{2a^4} \ln \left( \frac{x^2}{a^2 - x^2} \right) \\
175 \quad & \int \frac{dx}{x^2(a^2 - x^2)^2} = -\frac{1}{a^4 x} + \frac{x}{2a^4(a^2 - x^2)} + \frac{3}{4a^6} \ln \left( \frac{a+x}{a-x} \right) \\
176 \quad & \int \frac{dx}{x^3(a^2 - x^2)^2} = -\frac{1}{2a^4 x^2} + \frac{1}{2a^4(a^2 - x^2)} + \frac{1}{a^6} \ln \left( \frac{x^2}{a^2 - x^2} \right) \\
177 \quad & \int \frac{dx}{(a^2 - x^2)^n} = \frac{x}{2(n-1)a^2(a^2 - x^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(a^2 - x^2)^{n-1}} \\
178 \quad & \int \frac{x \, dx}{(a^2 - x^2)^n} = \frac{1}{2(n-1)(a^2 - x^2)^{n-1}} \\
179 \quad & \int \frac{dx}{x(a^2 - x^2)^n} = \frac{1}{2(n-1)a^2(a^2 - x^2)^{n-1}} + \frac{1}{a^2} \int \frac{dx}{x(a^2 - x^2)^{n-1}} \\
180 \quad & \int \frac{x^m \, dx}{(a^2 - x^2)^n} = a^2 \int \frac{x^{m-2} \, dx}{(a^2 - x^2)^n} - \int \frac{x^{m-2} \, dx}{(a^2 - x^2)^{n-1}} \\
181 \quad & \int \frac{dx}{x^m(a^2 - x^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^{m-2}(a^2 - x^2)^n} + \frac{1}{a^2} \int \frac{dx}{x^m(a^2 - x^2)^{n-1}}
\end{aligned}$$

#### INTEGRALES CON $\sqrt{x^2 + a^2}$

$$\begin{aligned}
182 \quad & \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( x + \sqrt{x^2 + a^2} \right) \quad \text{ó} \quad \operatorname{senh}^{-1} \frac{x}{a} \\
183 \quad & \int \frac{x \, dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2} \\
184 \quad & \int \frac{x^2 \, dx}{\sqrt{x^2 + a^2}} = \frac{x \sqrt{x^2 + a^2}}{2} - \frac{a^2}{2} \ln \left( x + \sqrt{x^2 + a^2} \right) \\
185 \quad & \int \frac{x^3 \, dx}{\sqrt{x^2 + a^2}} = \frac{\sqrt{(x^2 + a^2)^3}}{3} - a^2 \sqrt{x^2 + a^2} \\
186 \quad & \int \frac{dx}{x \sqrt{x^2 + a^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \\
187 \quad & \int \frac{dx}{x^2 \sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{a^2 x} \\
188 \quad & \int \frac{dx}{x^3 \sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{2a^2 x^2} + \frac{1}{2a^3} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \\
189 \quad & \int \sqrt{x^2 + a^2} \, dx = \frac{x \sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \ln \left( x + \sqrt{x^2 + a^2} \right)
\end{aligned}$$

$$|d| = \rho^2 \operatorname{sen} \theta$$

$$\begin{aligned}
190 \quad & \int x \sqrt{x^2 + a^2} dx = \frac{\sqrt{(x^2 + a^2)^3}}{3} \\
191 \quad & \int x^2 \sqrt{x^2 + a^2} dx = \frac{x \sqrt{(x^2 + a^2)^3}}{4} - \frac{a^2 x \sqrt{x^2 + a^2}}{8} - \frac{a^4}{8} \ln(x + \sqrt{x^2 + a^2}) \\
192 \quad & \int x^3 \sqrt{x^2 + a^2} dx = \frac{\sqrt{(x^2 + a^2)^5}}{5} - \frac{a^2 \sqrt{(x^2 + a^2)^3}}{3} \\
193 \quad & \int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \\
194 \quad & \int \frac{\sqrt{x^2 + a^2}}{x^2} dx = -\frac{\sqrt{x^2 + a^2}}{x} + \ln(x + \sqrt{x^2 + a^2}) \\
195 \quad & \int \frac{\sqrt{x^2 + a^2}}{x^3} dx = -\frac{\sqrt{x^2 + a^2}}{2x^2} - \frac{1}{2a} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \\
196 \quad & \int \frac{dx}{\sqrt{(x^2 + a^2)^3}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} \\
197 \quad & \int \frac{-x dx}{\sqrt{(x^2 + a^2)^3}} = \frac{-1}{\sqrt{x^2 + a^2}} \\
198 \quad & \int \frac{x^2 dx}{\sqrt{(x^2 + a^2)^3}} = \frac{-x}{\sqrt{x^2 + a^2}} + \ln(x + \sqrt{x^2 + a^2}) \\
199 \quad & \int \frac{x^3 dx}{\sqrt{(x^2 + a^2)^3}} = \sqrt{x^2 + a^2} + \frac{a^2}{\sqrt{x^2 + a^2}} \\
200 \quad & \int \frac{dx}{x \sqrt{(x^2 + a^2)^3}} = \frac{1}{a^2 \sqrt{x^2 + a^2}} - \frac{1}{a^3} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \\
201 \quad & \int \frac{dx}{x^2 \sqrt{(x^2 + a^2)^3}} = -\frac{\sqrt{x^2 + a^2}}{a^4 x} - \frac{x}{a^4 \sqrt{x^2 + a^2}} \\
202 \quad & \int \frac{dx}{x^3 \sqrt{(x^2 + a^2)^3}} = \frac{-1}{2a^2 x^2 \sqrt{x^2 + a^2}} - \frac{3}{2a^4 \sqrt{x^2 + a^2}} + \frac{3}{2a^5} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \\
203 \quad & \int \sqrt{(x^2 + a^2)^3} dx = \frac{x \sqrt{(x^2 + a^2)^3}}{4} + \frac{3a^2 x \sqrt{x^2 + a^2}}{8} + \frac{3a^4}{8} \ln(x + \sqrt{x^2 + a^2}) \\
204 \quad & \int x \sqrt{(x^2 + a^2)^3} dx = \frac{\sqrt{(x^2 + a^2)^5}}{5} \\
205 \quad & \int x^2 \sqrt{(x^2 + a^2)^3} dx = \frac{x \sqrt{(x^2 + a^2)^5}}{6} - \frac{a^2 x \sqrt{(x^2 + a^2)^3}}{24} - \frac{a^4 x \sqrt{x^2 + a^2}}{16} - \frac{a^6}{16} \ln(x + \sqrt{x^2 + a^2}) \\
206 \quad & \int x^3 \sqrt{(x^2 + a^2)^3} dx = \frac{\sqrt{(x^2 + a^2)^7}}{7} - \frac{a^2 \sqrt{(x^2 + a^2)^5}}{5} \\
207 \quad & \int \frac{\sqrt{(x^2 + a^2)^3}}{x} dx = \frac{\sqrt{(x^2 + a^2)^3}}{3} + a^2 \sqrt{x^2 + a^2} - a^3 \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right)
\end{aligned}$$

$$\begin{aligned}
208 \quad & \int \frac{\sqrt{(x^2 + a^2)^3}}{x^2} dx = -\frac{\sqrt{(x^2 + a^2)^3}}{x} + \frac{3x \sqrt{x^2 + a^2}}{2} + \frac{3}{2} a^2 \ln(x + \sqrt{x^2 + a^2}) \\
209 \quad & \int \frac{\sqrt{(x^2 + a^2)^3}}{x^3} dx = -\frac{\sqrt{(x^2 + a^2)^3}}{2x} + \frac{3 \sqrt{x^2 + a^2}}{2} - \frac{3a}{2} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right)
\end{aligned}$$

INTEGRALES CON  $\sqrt{x^2 - a^2}$

$$\begin{aligned}
210 \quad & \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) \\
211 \quad & \int \frac{x dx}{\sqrt{x^2 - a^2}} = \sqrt{x^2 - a^2} \\
212 \quad & \int \frac{x^2 dx}{\sqrt{x^2 - a^2}} = \frac{x \sqrt{x^2 - a^2}}{2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 - a^2}) \\
213 \quad & \int \frac{x^3 dx}{\sqrt{x^2 - a^2}} = \frac{\sqrt{(x^2 - a^2)^3}}{3} + a^2 \sqrt{x^2 - a^2} \\
214 \quad & \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| \\
215 \quad & \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} \\
216 \quad & \int \frac{dx}{x^3 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{2a^2 x^2} + \frac{1}{2a^3} \sec^{-1} \left| \frac{x}{a} \right| \\
217 \quad & \int \sqrt{x^2 - a^2} dx = \frac{x \sqrt{x^2 - a^2}}{2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 - a^2}) \\
218 \quad & \int x \sqrt{x^2 - a^2} dx = \frac{\sqrt{(x^2 - a^2)^3}}{3} \\
219 \quad & \int x^2 \sqrt{x^2 - a^2} dx = \frac{x \sqrt{(x^2 - a^2)^3}}{4} + \frac{a^2 x \sqrt{x^2 - a^2}}{8} - \frac{a^4}{8} \ln(x + \sqrt{x^2 - a^2}) \\
220 \quad & \int x^3 \sqrt{x^2 - a^2} dx = \frac{\sqrt{(x^2 - a^2)^5}}{5} + \frac{a^2 \sqrt{(x^2 - a^2)^3}}{3} \\
221 \quad & \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| \\
222 \quad & \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = -\frac{\sqrt{x^2 - a^2}}{x} + \ln(x + \sqrt{x^2 - a^2}) \\
223 \quad & \int \frac{\sqrt{x^2 - a^2}}{x^3} dx = -\frac{\sqrt{x^2 - a^2}}{2x^2} + \frac{1}{2a} \sec^{-1} \left| \frac{x}{a} \right| \\
224 \quad & \int \frac{dx}{\sqrt{(x^2 - a^2)^3}} = -\frac{x}{a^2 \sqrt{x^2 - a^2}}
\end{aligned}$$

$$225 \int \frac{x dx}{\sqrt{(x^2 - a^2)^3}} = \frac{-1}{\sqrt{x^2 - a^2}}$$

$$226 \int \frac{x^2 dx}{\sqrt{(x^2 - a^2)^3}} = \frac{-x}{\sqrt{x^2 - a^2}} + \ln \left( x + \sqrt{x^2 - a^2} \right)$$

$$227 \int \frac{x^3 dx}{\sqrt{(x^2 - a^2)^3}} = \sqrt{x^2 - a^2} - \frac{a^2}{\sqrt{x^2 - a^2}}$$

$$228 \int \frac{dx}{x \sqrt{(x^2 - a^2)^3}} = \frac{-1}{a^2 \sqrt{x^2 - a^2}} - \frac{1}{a^3} \sec^{-1} \left| \frac{x}{a} \right|$$

$$229 \int \frac{dx}{x^2 \sqrt{(x^2 - a^2)^3}} = -\frac{\sqrt{x^2 - a^2}}{a^4 x} - \frac{x}{a^4 \sqrt{x^2 - a^2}}$$

$$230 \int \frac{dx}{x^3 \sqrt{(x^2 - a^2)^3}} = \frac{1}{2 a^2 x^2 \sqrt{x^2 - a^2}} - \frac{3}{2 a^4 \sqrt{x^2 - a^2}} - \frac{3}{2 a^5} \sec^{-1} \left| \frac{x}{a} \right|$$

$$231 \int \sqrt{(x^2 - a^2)^3} dx = \frac{x \sqrt{(x^2 - a^2)^3}}{4} - \frac{3 a^2 x \sqrt{x^2 - a^2}}{8} + \frac{3 a^4}{8} \ln \left( x + \sqrt{x^2 - a^2} \right)$$

$$232 \int x \sqrt{(x^2 - a^2)^3} dx = \frac{\sqrt{(x^2 - a^2)^5}}{5}$$

$$233 \int x^2 \sqrt{(x^2 - a^2)^3} dx = \frac{x \sqrt{(x^2 - a^2)^5}}{6} + \frac{a^2 x \sqrt{(x^2 - a^2)^3}}{24} - \frac{a^4 x \sqrt{x^2 - a^2}}{16} + \frac{a^6}{16} \ln \left( x + \sqrt{x^2 - a^2} \right)$$

$$234 \int x^3 \sqrt{(x^2 - a^2)^3} dx = \frac{\sqrt{(x^2 - a^2)^7}}{7} + \frac{a^2 \sqrt{(x^2 - a^2)^5}}{5}$$

$$235 \int \frac{\sqrt{(x^2 - a^2)^3}}{x} dx = \frac{\sqrt{(x^2 - a^2)^3}}{3} - a^2 \sqrt{x^2 - a^2} + a^3 \sec^{-1} \left| \frac{x}{a} \right|$$

$$236 \int \frac{\sqrt{(x^2 - a^2)^3}}{x^2} dx = -\frac{\sqrt{(x^2 - a^2)^3}}{x} + \frac{3 x \sqrt{x^2 - a^2}}{2} - \frac{3}{2} a^2 \ln \left( x + \sqrt{x^2 - a^2} \right)$$

$$237 \int \frac{\sqrt{(x^2 - a^2)^3}}{x^3} dx = -\frac{\sqrt{(x^2 - a^2)^3}}{2 x^2} + \frac{3 \sqrt{x^2 - a^2}}{2} - \frac{3}{2} a \sec^{-1} \left| \frac{x}{a} \right|$$

#### INTEGRALES CON $\sqrt{a^2 - x^2}$

$$238 \int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{sen}^{-1} \frac{x}{a}$$

$$239 \int \frac{x dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$$

$$240 \int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x \sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{x}{a}$$

$$241 \int \frac{x^3 dx}{\sqrt{a^2 - x^2}} = \frac{\sqrt{(a^2 - x^2)^3}}{3} - a^2 \sqrt{a^2 - x^2}$$

$$242 \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

$$243 \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x}$$

$$244 \int \frac{dx}{x^3 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{2 a^2 x^2} - \frac{1}{2 a^3} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

$$245 \int \sqrt{a^2 - x^2} dx = \frac{x \sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{x}{a}$$

$$246 \int x \sqrt{a^2 - x^2} dx = -\frac{\sqrt{(a^2 - x^2)^3}}{3}$$

$$247 \int x^2 \sqrt{a^2 - x^2} dx = -\frac{x \sqrt{(a^2 - x^2)^3}}{4} + \frac{a^2 x \sqrt{a^2 - x^2}}{8} - \frac{a^4}{8} \operatorname{sen}^{-1} \frac{x}{a}$$

$$248 \int x^3 \sqrt{a^2 - x^2} dx = \frac{\sqrt{(a^2 - x^2)^5}}{5} - \frac{a^2 \sqrt{(a^2 - x^2)^3}}{3}$$

$$249 \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

$$250 \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \operatorname{sen}^{-1} \frac{x}{a}$$

$$251 \int \frac{\sqrt{a^2 - x^2}}{x^3} dx = -\frac{\sqrt{a^2 - x^2}}{2 x^2} + \frac{1}{2 a} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

$$252 \int \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{a^2 \sqrt{a^2 - x^2}}$$

$$253 \int \frac{x dx}{\sqrt{(a^2 - x^2)^3}} = \frac{1}{\sqrt{a^2 - x^2}}$$

$$254 \int \frac{x^2 dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x^2}} - \operatorname{sen}^{-1} \frac{x}{a}$$

$$255 \int \frac{x^3 dx}{\sqrt{(a^2 - x^2)^3}} = \sqrt{a^2 - x^2} + \frac{a^2}{\sqrt{a^2 - x^2}}$$

$$256 \int \frac{dx}{x \sqrt{(a^2 - x^2)^3}} = \frac{1}{a^2 \sqrt{a^2 - x^2}} - \frac{1}{a^3} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

$$257 \int \frac{dx}{x^2 \sqrt{(a^2 - x^2)^3}} = -\frac{\sqrt{a^2 - x^2}}{a^4 x} + \frac{x}{a^4 \sqrt{a^2 - x^2}}$$

$$258 \int \frac{dx}{x^3 \sqrt{(a^2 - x^2)^3}} = \frac{-1}{2 a^2 x^2 \sqrt{a^2 - x^2}} + \frac{3}{2 a^4 \sqrt{a^2 - x^2}} - \frac{3}{2 a^5} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)$$

CG  $\rightarrow$  Problema D  $\in \mathbb{R}^2$

$$\begin{aligned}
259 \int \sqrt{a^2 - x^2}^3 dx &= \frac{x \sqrt{a^2 - x^2}^3}{4} + \frac{3 a^2 x \sqrt{a^2 - x^2}}{8} + \frac{3 a^4 \operatorname{sen}^{-1} \frac{x}{a}}{8} \\
260 \int x \sqrt{a^2 - x^2}^3 dx &= -\frac{\sqrt{a^2 - x^2}^5}{5} \\
261 \int x^2 \sqrt{a^2 - x^2}^3 dx &= -\frac{x \sqrt{a^2 - x^2}^5}{6} + \frac{a^2 x \sqrt{a^2 - x^2}^3}{24} + \frac{a^4 x \sqrt{a^2 - x^2}}{16} + \frac{a^6 \operatorname{sen}^{-1} \frac{x}{a}}{16} \\
262 \int x^3 \sqrt{a^2 - x^2}^3 dx &= \frac{\sqrt{a^2 - x^2}^7}{7} - \frac{a^2 \sqrt{a^2 - x^2}^5}{5} \\
263 \int \frac{\sqrt{a^2 - x^2}^3}{x} dx &= \frac{\sqrt{a^2 - x^2}^3}{3} + a^2 \sqrt{a^2 - x^2} - a^3 \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) \\
264 \int \frac{\sqrt{a^2 - x^2}}{x^2} dx &= -\frac{\sqrt{a^2 - x^2}}{x} - \frac{3 x \sqrt{a^2 - x^2}}{2} - \frac{3}{2} a^2 \operatorname{sen}^{-1} \frac{x}{a} \\
265 \int \frac{\sqrt{a^2 - x^2}}{x^3} dx &= -\frac{\sqrt{a^2 - x^2}}{2 x^2} - \frac{3 \sqrt{a^2 - x^2}}{2} + \frac{3}{2} a \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)
\end{aligned}$$

#### INTEGRALES CON $ax^2 + bx + c$

Si  $b^2 = 4ac$ , se puede escribir  $ax^2 + bx + c = a \left( x + \frac{b}{2a} \right)^2$  y se emplean los resultados de las páginas 11 y 12.

$$\begin{aligned}
266 \int \frac{dx}{ax^2 + bx + c} &= \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \operatorname{tg}^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} \\ \frac{1}{\sqrt{b^2 - 4ac}} \ln \left( \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right) \end{cases} \\
267 \int \frac{x dx}{ax^2 + bx + c} &= \frac{1}{2a} \ln(ax^2 + bx + c) - \frac{b}{2a} \int \frac{dx}{ax^2 + bx + c} \\
268 \int \frac{x^2 dx}{ax^2 + bx + c} &= \frac{x}{a} - \frac{b}{2a^2} \ln(ax^2 + bx + c) + \frac{b^2 - 2ac}{2a^2} \int \frac{dx}{ax^2 + bx + c} \\
269 \int \frac{x^n dx}{ax^2 + bx + c} &= \frac{x^{n-1}}{(n-1)a} - \frac{c}{a} \int \frac{x^{n-2} dx}{ax^2 + bx + c} - \frac{b}{a} \int \frac{x^{n-1} dx}{ax^2 + bx + c} \\
270 \int \frac{dx}{x(ax^2 + bx + c)} &= \frac{1}{2c} \ln \left( \frac{x^2}{ax^2 + bx + c} \right) - \frac{b}{2c} \int \frac{dx}{ax^2 + bx + c} \\
271 \int \frac{dx}{x^2(ax^2 + bx + c)} &= \frac{b}{2c^2} \ln \left( \frac{ax^2 + bx + c}{x^2} \right) - \frac{1}{x^2 c} + \frac{b^2 - 2ac}{2c^2} \int \frac{dx}{ax^2 + bx + c} \\
272 \int \frac{dx}{x^n(ax^2 + bx + c)} &= -\frac{1}{(n-1)c x^{n-1}} - \frac{b}{c} \int \frac{dx}{x^{n-1}(ax^2 + bx + c)} - \frac{a}{c} \int \frac{dx}{x^{n-2}(ax^2 + bx + c)} \\
273 \int \frac{dx}{(ax^2 + bx + c)^2} &= \frac{2ax + b}{(4ac - b^2)(ax^2 + bx + c)} + \frac{2a}{4ac - b^2} \int \frac{dx}{ax^2 + bx + c}
\end{aligned}$$

$$\begin{aligned}
274 \int \frac{x dx}{(ax^2 + bx + c)^2} &= -\frac{bx + 2c}{(4ac - b^2)(ax^2 + bx + c)} - \frac{b}{4ac - b^2} \int \frac{dx}{ax^2 + bx + c} \\
275 \int \frac{x^2 dx}{(ax^2 + bx + c)^2} &= \frac{(b^2 - 2ac)x + bc}{a(4ac - b^2)(ax^2 + bx + c)} + \frac{2c}{4ac - b^2} \int \frac{dx}{ax^2 + bx + c} \\
276 \int \frac{x^n dx}{(ax^2 + bx + c)^n} &= \frac{-x^{n-1}}{(2n-m-1)a(ax^2 + bx + c)^{n-1}} + \frac{1}{(2n-m-1)a} \left\{ c \int \frac{(m-1)x^{m-2} dx}{(ax^2 + bx + c)^n} - b \int \frac{(n-m)x^{m-1} dx}{(ax^2 + bx + c)^n} \right\} \\
277 \int \frac{x^{2n-1} dx}{(ax^2 + bx + c)^n} &= \frac{1}{a} \int \frac{x^{2n-3} dx}{(ax^2 + bx + c)^{n-1}} - \frac{c}{a} \int \frac{x^{2n-3} dx}{(ax^2 + bx + c)^n} - \frac{b}{a} \int \frac{x^{2n-2} dx}{(ax^2 + bx + c)^n} \\
278 \int \frac{dx}{x(ax^2 + bx + c)^2} &= \frac{1}{2c(ax^2 + bx + c)} - \frac{b}{2c} \int \frac{dx}{(ax^2 + bx + c)^2} + \frac{1}{c} \int \frac{dx}{x(ax^2 + bx + c)} \\
279 \int \frac{dx}{x^2(ax^2 + bx + c)^2} &= \frac{-1}{cx(ax^2 + bx + c)} - \frac{3a}{c} \int \frac{dx}{(ax^2 + bx + c)^2} - \frac{2b}{c} \int \frac{dx}{x(ax^2 + bx + c)} \\
280 \int \frac{dx}{x^m(ax^2 + bx + c)^n} &= \frac{-1}{(m-1)c x^{m-1}(ax^2 + bx + c)^n} \\
&\quad - \frac{(m+2n-3)a}{(m-1)c} \int \frac{dx}{x^{m-2}(ax^2 + bx + c)^n} \\
&\quad - \frac{(m+n-2)b}{(m-1)c} \int \frac{dx}{x^{m-1}(ax^2 + bx + c)^n}
\end{aligned}$$

#### INTEGRALES CON $\sqrt{ax^2 + bx + c}$

Si en las fórmulas siguientes  $b^2 = 4ac$ , se puede escribir  $\sqrt{ax^2 + bx + c} = \sqrt{a} \left( x + \frac{b}{2a} \right)$  y se emplean los resultados de las páginas 11 y 12.

$$\begin{aligned}
281 \int \frac{dx}{\sqrt{ax^2 + bx + c}} &= \begin{cases} \frac{1}{\sqrt{a}} \ln \left( 2\sqrt{a} \sqrt{ax^2 + bx + c} + 2ax + b \right) \\ -\frac{1}{\sqrt{-a}} \operatorname{sen}^{-1} \left( \frac{2ax + b}{\sqrt{b^2 - 4ac}} \right) \quad \text{ó} \quad \frac{1}{\sqrt{a}} \operatorname{sen}^{-1} \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right) \end{cases} \\
282 \int \frac{x dx}{\sqrt{ax^2 + bx + c}} &= \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2 + bx + c}} \\
283 \int \frac{x^2 dx}{\sqrt{ax^2 + bx + c}} &= \frac{2ax - 3b}{4a^2} \sqrt{ax^2 + bx + c} + \frac{3b^2 - 4ac}{8a^2} \int \frac{dx}{\sqrt{ax^2 + bx + c}} \\
284 \int \frac{dx}{x \sqrt{ax^2 + bx + c}} &= \begin{cases} -\frac{1}{\sqrt{c}} \ln \left( \frac{2\sqrt{c} \sqrt{ax^2 + bx + c} + bx + 2c}{x} \right) \\ \frac{1}{\sqrt{-c}} \operatorname{sen}^{-1} \left( \frac{bx + 2c}{|x| \sqrt{b^2 - 4ac}} \right) \quad \text{ó} \quad -\frac{1}{\sqrt{c}} \operatorname{sen}^{-1} \left( \frac{bx + 2c}{|x| \sqrt{4ac - b^2}} \right) \end{cases} \\
285 \int \frac{dx}{x^2 \sqrt{ax^2 + bx + c}} &= -\frac{\sqrt{ax^2 + bx + c}}{cx} - \frac{b}{2c} \int \frac{dx}{x \sqrt{ax^2 + bx + c}} \\
286 \int \sqrt{ax^2 + bx + c} dx &= \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}
\end{aligned}$$

$$287 \int x \sqrt{ax^2 + bx + c} dx = \frac{\sqrt{(ax^2+bx+c)^3}}{3a} - \frac{b(2ax+b)}{8a^2} \sqrt{ax^2+bx+c} - \frac{b(4ac-b^2)}{16a^2} \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$288 \int x^2 \sqrt{ax^2 + bx + c} dx = \frac{8ax - 5b}{24a^2} \sqrt{(ax^2 + bx + c)^3} + \frac{5b^2 - 4ac}{16a^2} \int \sqrt{ax^2 + bx + c} dx$$

$$289 \int \frac{\sqrt{ax^2 + bx + c}}{x} dx = \sqrt{ax^2 + bx + c} + \frac{b}{2} \int \frac{dx}{\sqrt{ax^2 + bx + c}} + c \int \frac{dx}{x \sqrt{ax^2 + bx + c}}$$

$$290 \int \frac{\sqrt{ax^2 + bx + c}}{x^2} dx = -\frac{\sqrt{ax^2 + bx + c}}{x} + a \int \frac{dx}{\sqrt{ax^2 + bx + c}} + \frac{b}{2} \int \frac{dx}{x \sqrt{ax^2 + bx + c}}$$

$$291 \int \frac{dx}{\sqrt{(ax^2 + bx + c)^3}} = \frac{2(2ax + b)}{(4ac - b^2) \sqrt{ax^2 + bx + c}}$$

$$292 \int \frac{x dx}{\sqrt{(ax^2 + bx + c)^3}} = \frac{2(bx + 2c)}{(b^2 - 4ac) \sqrt{ax^2 + bx + c}}$$

$$293 \int \frac{x^2 dx}{\sqrt{(ax^2 + bx + c)^3}} = \frac{(2b^2 - 4ac)x + 2bc}{a(4ac - b^2) \sqrt{ax^2 + bx + c}} + \frac{1}{a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$294 \int \frac{dx}{x \sqrt{(ax^2 + bx + c)^3}} = \frac{1}{c \sqrt{ax^2 + bx + c}} + \frac{1}{c} \int \frac{dx}{x \sqrt{ax^2 + bx + c}} - \frac{b}{2c} \int \frac{dx}{\sqrt{(ax^2 + bx + c)^3}}$$

$$295 \int \frac{dx}{x^2 \sqrt{(ax^2 + bx + c)^3}} = \frac{-(ax^2 + bx + c)}{c^2 x \sqrt{ax^2 + bx + c}} - \frac{3b}{2c^2} \int \frac{dx}{x \sqrt{ax^2 + bx + c}} + \frac{b^2 - 4ac}{2c^2} \int \frac{dx}{\sqrt{(ax^2 + bx + c)^3}}$$

$$296 \int \sqrt{(ax^2 + bx + c)^{n+1}} dx = \frac{(2ax+b)\sqrt{(ax^2+bx+c)^{n+1}}}{4a(n+1)} + \frac{(2n+1)(4ac-b^2)}{8a(n+1)} \int \sqrt{(ax^2+bx+c)^{n-1}} dx$$

$$297 \int x \sqrt{(ax^2 + bx + c)^{n+1}} dx = \frac{\sqrt{(ax^2 + bx + c)^{n+3}}}{a(2n+3)} - \frac{b}{2a} \int \sqrt{(ax^2 + bx + c)^{n+1}} dx$$

$$298 \int \frac{dx}{\sqrt{(ax^2 + bx + c)^{n+1}}} = \frac{1}{(2n-1)(4ac-b^2)} \left( \frac{2(2ax+b)}{\sqrt{(ax^2+bx+c)^{n-1}}} + 8a(n-1) \int \frac{dx}{\sqrt{(ax^2+bx+c)^{n-1}}} \right)$$

$$299 \int \frac{dx}{\sqrt{(ax^2 + bx + c)^{n+1}}} = \frac{1}{(2n-1)c \sqrt{(ax^2+bx+c)^{n-1}}} + \frac{1}{c} \int \frac{dx}{x \sqrt{(ax^2+bx+c)^{n-1}}} - \frac{b}{2c} \int \frac{dx}{\sqrt{(ax^2+bx+c)^{n+1}}}$$

### INTEGRALES CON $x^3 + a^3$

Para integrales con  $x^3 - a^3$ , se reemplaza  $a$  por  $-a$

$$300 \int \frac{dx}{x^3 + a^3} = \frac{1}{6a^2} \ln \frac{(x+a)^2}{x^2 - ax + a^2} + \frac{1}{a^2 \sqrt{3}} \operatorname{tg}^{-1} \frac{2x-a}{a \sqrt{3}}$$

$$301 \int \frac{x dx}{x^3 + a^3} = \frac{1}{6a} \ln \frac{x^2 - ax + a^2}{(x+a)^2} + \frac{1}{a \sqrt{3}} \operatorname{tg}^{-1} \frac{2x-a}{a \sqrt{3}}$$

$$302 \int \frac{x^2 dx}{x^3 + a^3} = \frac{1}{3} \ln(x^3 + a^3)$$

$$303 \int \frac{dx}{x(x^3 + a^3)} = \frac{1}{3a^3} \ln \left( \frac{x^3}{x^3 + a^3} \right)$$

$$304 \int \frac{dx}{x^2(x^3 + a^3)} = -\frac{1}{a^3 x} - \frac{1}{6a^4} \ln \frac{x^2 - ax + a^2}{(x+a)^2} - \frac{1}{a^4 \sqrt{3}} \operatorname{tg}^{-1} \frac{2x-a}{a \sqrt{3}}$$

$$305 \int \frac{dx}{(x^3 + a^3)^2} = \frac{x}{3a^3(x^3 + a^3)} + \frac{1}{9a^6} \ln \frac{(x+a)^2}{x^2 - ax + a^2} + \frac{2}{3a^6 \sqrt{3}} \operatorname{tg}^{-1} \frac{2x-a}{a \sqrt{3}}$$

$$306 \int \frac{x dx}{(x^3 + a^3)^2} = \frac{x^2}{3a^3(x^3 + a^3)} + \frac{1}{18a^4} \ln \frac{x^2 - ax + a^2}{(x+a)^2} + \frac{1}{3a^4 \sqrt{3}} \operatorname{tg}^{-1} \frac{2x-a}{a \sqrt{3}}$$

$$307 \int \frac{x^2 dx}{(x^3 + a^3)^2} = -\frac{1}{3(x^3 + a^3)}$$

$$308 \int \frac{dx}{x(x^3 + a^3)^2} = \frac{1}{3a^3(x^3 + a^3)} + \frac{1}{3a^6} \ln \left( \frac{x^3}{x^3 + a^3} \right)$$

$$309 \int \frac{dx}{x^2(x^3 + a^3)^2} = -\frac{1}{a^6 x} - \frac{1}{3a^6(x^3 + a^3)} - \frac{4}{3a^6} \int \frac{x dx}{x^3 + a^3} \quad \text{Véase 301}$$

$$310 \int \frac{x^m dx}{x^3 + a^3} = \frac{x^{m-2}}{(m-2)} - a^3 \int \frac{x^{m-3} dx}{x^3 + a^3}$$

$$311 \int \frac{dx}{x^n(x^3 + a^3)} = \frac{-1}{a^3(n-1)x^{n-1}} - \frac{1}{a^3} \int \frac{dx}{x^{n-3}(x^3 + a^3)}$$

### INTEGRALES CON $x^4 \pm a^4$

$$312 \int \frac{dx}{x^4 + a^4} = \frac{1}{4a^3 \sqrt{2}} \ln \left( \frac{x^2 + ax \sqrt{2 + a^2}}{x^2 - ax \sqrt{2 + a^2}} \right) - \frac{1}{2a^3 \sqrt{2}} \operatorname{tg}^{-1} \frac{ax \sqrt{2}}{x^2 - a^2}$$

$$313 \int \frac{x dx}{x^4 + a^4} = \frac{1}{2a^2} \operatorname{tg}^{-1} \frac{x^2}{a^2}$$

$$314 \int \frac{x^2 dx}{x^4 + a^4} = \frac{1}{4a \sqrt{2}} \ln \left( \frac{x^2 - ax \sqrt{2 + a^2}}{x^2 + ax \sqrt{2 + a^2}} \right) - \frac{1}{2a \sqrt{2}} \operatorname{tg}^{-1} \frac{ax \sqrt{2}}{x^2 - a^2}$$

$$315 \int \frac{x^3 dx}{x^4 + a^4} = \frac{1}{4} \ln(x^4 + a^4)$$

$$316 \int \frac{dx}{x(x^4 + a^4)} = \frac{1}{4a^4} \ln \left( \frac{x^4}{x^4 + a^4} \right)$$

$$317 \int \frac{dx}{x^2(x^4 + a^4)} = -\frac{1}{a^4 x} - \frac{1}{4a^6 \sqrt{2}} \ln \left( \frac{x^2 - ax \sqrt{2 + a^2}}{x^2 + ax \sqrt{2 + a^2}} \right) + \frac{1}{2a^6 \sqrt{2}} \operatorname{tg}^{-1} \frac{ax \sqrt{2}}{x^2 - a^2}$$

$$318 \int \frac{dx}{x^3(x^4 + a^4)} = -\frac{1}{2a^4 x^2} - \frac{1}{2a^6} \operatorname{tg}^{-1} \frac{x^2}{a^2}$$

$$319 \int \frac{dx}{x^4 - a^4} = \frac{1}{4a^3} \ln \left( \frac{x-a}{x+a} \right) - \frac{1}{2a^3} \operatorname{tg}^{-1} \frac{x}{a}$$

$$320 \int \frac{x dx}{x^4 - a^4} = \frac{1}{4a^2} \ln \left( \frac{x^2 - a^2}{x^2 + a^2} \right)$$

$$321 \int \frac{x^2 dx}{x^4 - a^4} = \frac{1}{4a} \ln \left( \frac{x-a}{x+a} \right) + \frac{1}{2a} \operatorname{tg}^{-1} \frac{x}{a}$$

$$322 \int \frac{x^3 dx}{x^4 - a^4} = \frac{1}{4} \ln(x^4 - a^4)$$

$$323 \int \frac{dx}{x(x^4 - a^4)} = \frac{1}{4a^4} \ln\left(\frac{x^4 - a^4}{x^4}\right)$$

$$324 \int \frac{dx}{x^2(x^4 - a^4)} = \frac{1}{a^4 x} + \frac{1}{4a^6} \ln\left(\frac{x-a}{x+a}\right) + \frac{1}{2a^6} \operatorname{tg}^{-1} \frac{x}{a}$$

$$325 \int \frac{dx}{x^3(x^4 - a^4)} = \frac{1}{2a^4 x^2} + \frac{1}{4a^6} \ln\left(\frac{x^2 - a^2}{x^2 + a^2}\right)$$

#### INTEGRALES CON $x^n \pm a^n$

$$326 \int \frac{dx}{x(x^n + a^n)} = \frac{1}{n a^n} \ln \frac{x^n}{x^n + a^n}$$

$$327 \int \frac{x^{n-1} dx}{x^n + a^n} = \frac{1}{n} \ln(x^n + a^n)$$

$$328 \int \frac{x^m dx}{(x^n + a^n)^p} = \int \frac{x^{m-r} dx}{(x^n + a^n)^{p-1}} - a^n \int \frac{x^{m-r} dx}{(x^n + a^n)^p}$$

$$329 \int \frac{dx}{x^m (x^n + a^n)^p} = \frac{1}{a^n} \int \frac{dx}{x^m (x^n + a^n)^{p-1}} - \frac{1}{a^n} \int \frac{dx}{x^{m-r} (x^n + a^n)^p}$$

$$330 \int \frac{dx}{x \sqrt{x^n + a^n}} = \frac{1}{n \sqrt{a^n}} \ln \left( \frac{\sqrt{x^n + a^n} - \sqrt{a^n}}{\sqrt{x^n + a^n} + \sqrt{a^n}} \right)$$

$$331 \int \frac{dx}{x(x^n - a^n)} = \frac{1}{n a^n} \ln \left( \frac{x^n - a^n}{x^n} \right)$$

$$332 \int \frac{x^{n-1} dx}{x^n - a^n} = \frac{1}{n} \ln(x^n - a^n)$$

$$333 \int \frac{x^m dx}{(x^n - a^n)^p} = a^n \int \frac{x^{m-r} dx}{(x^n - a^n)^{p-1}} + \int \frac{x^{m-r} dx}{(x^n - a^n)^p}$$

$$334 \int \frac{dx}{x^m (x^n - a^n)^p} = \frac{1}{a^n} \int \frac{dx}{x^m (x^n - a^n)^{p-1}} - \frac{1}{a^n} \int \frac{dx}{x^{m-r} (x^n - a^n)^p}$$

$$335 \int \frac{dx}{x \sqrt{x^n - a^n}} = \frac{2}{n \sqrt{a^n}} \cos^{-1} \sqrt{\frac{a^n}{x^n}}$$

$$336 \int \frac{x^{p-1} dx}{(x^{2m} + a^{2m})} = \frac{1}{m a^{2m-p}} \sum_{k=1}^m \operatorname{sen} \left( \frac{(2k-1)p\pi}{2m} \right) \cdot \operatorname{tg}^{-1} \left( \frac{x + a \cos \left( \frac{(2k-1)\pi}{2m} \right)}{a \operatorname{sen} \left( \frac{(2k-1)\pi}{2m} \right)} \right) \\ - \frac{1}{2m a^{2m-p}} \sum_{k=1}^m \cos \left( \frac{(2k-1)p\pi}{2m} \right) \cdot \ln \{ x^2 + 2ax \cos \left( \frac{(2k-1)\pi}{2m} \right) + a^2 \}$$

$$337 \int \frac{x^{p-1} dx}{(x^{2m} - a^{2m})} = \frac{1}{2m a^{2m-p}} \sum_{k=1}^{m-1} \cos \frac{k p \pi}{m} \cdot \ln \{ x^2 - 2ax \cos \frac{k \pi}{m} + a^2 \} \\ - \frac{1}{m a^{2m-p}} \sum_{k=1}^{m-1} \operatorname{sen} \frac{k p \pi}{m} \cdot \operatorname{tg}^{-1} \left( \frac{x - a \cos \frac{k \pi}{m}}{a \operatorname{sen} \frac{k \pi}{m}} \right) \\ + \frac{1}{2m a^{2m-p}} \{ \ln(x-a) + (-1)^p \ln(x+a) \}$$

tanto en 336 como en 337 es  $0 < p \leq 2m$ .

$$338 \int \frac{x^{p-1} dx}{x^{2m+1} + a^{2m+1}} = \frac{2(-1)^{p-1}}{(2m+1) a^{2m-p+1}} \sum_{k=1}^m \operatorname{sen} \frac{2kp\pi}{2m+1} \cdot \operatorname{tg}^{-1} \left( \frac{x + a \cos \frac{2k\pi}{2m+1}}{a \operatorname{sen} \frac{2k\pi}{2m+1}} \right) \\ - \frac{(-1)^{p-1}}{(2m+1) a^{2m-p+1}} \sum_{k=1}^m \cos \frac{2kp\pi}{2m+1} \cdot \ln \{ x^2 - 2ax \cos \frac{2k\pi}{2m+1} + a^2 \} \\ + \frac{(-1)^{p-1} \ln(x+a)}{(2m+1) a^{2m-p+1}}$$

$$339 \int \frac{x^{p-1} dx}{x^{2m+1} - a^{2m+1}} = \frac{-2}{(2m+1) a^{2m-p+1}} \sum_{k=1}^m \operatorname{sen} \frac{2kp\pi}{2m+1} \cdot \operatorname{tg}^{-1} \left( \frac{x - a \cos \frac{2k\pi}{2m+1}}{a \operatorname{sen} \frac{2k\pi}{2m+1}} \right) \\ + \frac{1}{(2m+1) a^{2m-p+1}} \sum_{k=1}^m \cos \frac{2kp\pi}{2m+1} \cdot \ln \{ x^2 - 2ax \cos \frac{2k\pi}{2m+1} + a^2 \} \\ + \frac{\ln(x-a)}{(2m+1) a^{2m-p+1}}$$

tanto en 338 como en 339 es  $0 < p \leq 2m+1$ .

#### INTEGRALES CON $\operatorname{sen} ax$

$$340 \int \operatorname{sen} ax dx = -\frac{\cos ax}{a}$$

$$341 \int x \operatorname{sen} ax dx = \frac{\operatorname{sen} ax}{a^2} - \frac{x \cos ax}{a}$$

$$342 \int x^2 \operatorname{sen} ax dx = \frac{2x}{a^2} \operatorname{sen} ax + \left( \frac{2}{a^3} - \frac{x^2}{a} \right) \cos ax$$

$$343 \int x^3 \operatorname{sen} ax dx = \left( \frac{3x^2}{a^2} - \frac{6}{a^4} \right) \operatorname{sen} ax + \left( \frac{6x}{a^3} - \frac{x^3}{a} \right) \cos ax$$

$$344 \int \frac{\operatorname{sen} ax}{x} dx = ax - \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (ax)^{2n-1}}{(2n-1)(2n-1)!}$$

$$345 \int \frac{\operatorname{sen} ax}{x^2} dx = -\frac{\operatorname{sen} ax}{x} + a \int \frac{\cos ax}{x} dx \quad \text{Véase 374}$$

$$346 \int \frac{dx}{\operatorname{sen} ax} = \frac{1}{a} \ln(\operatorname{cosec} ax - \cotg ax) = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2}$$

$$347 \int \frac{x dx}{\operatorname{sen} ax} = \frac{1}{a^2} \left\{ ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \dots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \dots \right\} \quad B_n \text{ es}$$



nº de Bernoulli

$$348 \int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$349 \int x \sin^2 ax \, dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$$

$$350 \int \sin^3 ax \, dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a}$$

$$351 \int \sin^4 ax \, dx = \frac{3x}{8} - \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$$

$$352 \int \frac{dx}{\sin^2 ax} = -\frac{1}{a} \cotg ax$$

$$353 \int \frac{dx}{\sin^3 ax} = -\frac{\cos ax}{2a \sin^2 ax} + \frac{1}{2a} \ln \tg \frac{ax}{2}$$

$$354 \int \sin px \sin qx \, dx = \frac{\sin(p-q)x}{2(p-q)} - \frac{\sin(p+q)x}{2(p+q)}$$

Si  $p = \pm q$ , véase 348  $\Leftrightarrow p \neq q$  ojo

$$355 \int \frac{dx}{1 - \sin ax} = \frac{1}{a} \tg \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$356 \int \frac{x \, dx}{1 - \sin ax} = \frac{x}{a} \tg \left( \frac{\pi}{4} + \frac{ax}{2} \right) + \frac{2}{a^2} \ln \sin \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$357 \int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tg \left( \frac{\pi}{4} - \frac{ax}{2} \right)$$

$$358 \int \frac{x \, dx}{1 + \sin ax} = -\frac{x}{a} \tg \left( \frac{\pi}{4} - \frac{ax}{2} \right) + \frac{2}{a^2} \ln \sin \left( \frac{\pi}{4} - \frac{ax}{2} \right)$$

$$359 \int \frac{dx}{(1 - \sin ax)^2} = \frac{1}{2a} \tg \left( \frac{\pi}{4} + \frac{ax}{2} \right) + \frac{1}{6a} \tg^3 \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$360 \int \frac{dx}{(1 + \sin ax)^2} = -\frac{1}{2a} \tg \left( \frac{\pi}{4} - \frac{ax}{2} \right) - \frac{1}{6a} \tg^3 \left( \frac{\pi}{4} - \frac{ax}{2} \right)$$

$$361 \int \frac{dx}{p + q \sin ax} = \begin{cases} \frac{2}{a \sqrt{p^2 - q^2}} \tg^{-1} \frac{p \tg \frac{ax}{2} + q}{\sqrt{p^2 - q^2}} \\ \frac{1}{a \sqrt{q^2 - p^2}} \ln \left( \frac{p \tg \frac{ax}{2} + q - \sqrt{q^2 - p^2}}{p \tg \frac{ax}{2} + q + \sqrt{q^2 - p^2}} \right) \end{cases}$$

Si  $p = \pm q$ ,

véanse 355 y 357

$$362 \int \frac{dx}{(p + q \sin ax)^2} = \frac{q \cos ax}{a(p^2 - q^2)(p + q \sin ax)} + \frac{p}{(p^2 - q^2)} \int \frac{dx}{p + q \sin ax}$$

Si  $p = \pm q$ , véanse 359 y 360

$$363 \int \frac{dx}{p^2 + q^2 \sin^2 ax} = \frac{1}{a p \sqrt{p^2 + q^2}} \tg^{-1} \frac{\sqrt{p^2 + q^2} \cdot \tg ax}{p}$$

Ty es diferenciable hasta 10, don me 1 en el E de (1/n)  $E(1/n) \in S$

$$364 \int \frac{dx}{p^2 - q^2 \sin^2 ax} = \begin{cases} \frac{1}{a p \sqrt{p^2 - q^2}} \tg^{-1} \frac{p - q \cdot \tg ax}{p} \\ \frac{1}{2 a p \sqrt{q^2 - p^2}} \ln \left( \frac{\sqrt{q^2 - p^2} \cdot \tg ax + p}{\sqrt{q^2 - p^2} \cdot \tg ax - p} \right) \end{cases}$$

$$365 \int x^m \sin ax \, dx = -\frac{x^m \cos ax}{a} + \frac{m x^{m-1} \sin ax}{a^2} - \frac{m(m-1)}{a^2} \int x^{m-2} \sin ax \, dx$$

$$366 \int \frac{\sin ax}{x^n} \, dx = -\frac{\sin ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\cos ax}{x^{n-1}} \, dx \text{ Véase 396}$$

$$367 \int \sin^n ax \, dx = -\frac{\sin^{n-1} ax \cos ax}{a n} + \frac{n-1}{n} \int \sin^{n-2} ax \, dx$$

$$368 \int \frac{dx}{\sin^n ax} = \frac{-\cos ax}{a(n-1)\sin^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\sin^{n-2} ax}$$

$$369 \int \frac{x \, dx}{\sin^n ax} = \frac{-x \cos ax}{a(n-1)\sin^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\sin^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x \, dx}{\sin^{n-2} ax}$$

# INTEGRALES CON $\cos ax$

$$370 \int \cos ax \, dx = \frac{\sin ax}{a}$$

$$371 \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$372 \int x^2 \cos ax \, dx = \frac{2x}{a^2} \cos ax + \left( \frac{x^2}{a} - \frac{2}{a^2} \right) \sin ax$$

$$373 \int x^3 \cos ax \, dx = \left( \frac{3x^2}{a^2} - \frac{6}{a^2} \right) \cos ax + \left( \frac{x^3}{a} - \frac{6x}{a^2} \right) \sin ax$$

$$374 \int \frac{\cos ax}{x} \, dx = \ln x - \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} - \frac{(ax)^6}{6 \cdot 6!} + \dots = \ln x + \sum_{n=1}^{\infty} \frac{(-1)^n (ax)^{2n}}{(2n) \cdot (2n)!}$$

$$375 \int \frac{\cos ax}{x^2} \, dx = -\frac{\cos ax}{x} - a \int \frac{\sin ax}{x} \, dx \text{ Véase 374}$$

$$376 \int \frac{dx}{\cos ax} = \frac{1}{a} \ln(\sec ax + \tg ax) = \frac{1}{a} \ln \tg \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$377 \int \frac{x \, dx}{\cos ax} = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \dots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\} \quad E_n \text{ es nº de Euler}$$

$$378 \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$379 \int x \cos^2 ax \, dx = \frac{x^2}{4} + \frac{x \sin 2ax}{4a} + \frac{\cos 2ax}{8a^2}$$

$$380 \int \cos^3 ax \, dx = \frac{\sin ax}{a} - \frac{\sin^3 ax}{3a}$$

$$381 \int \cos^4 ax \, dx = \frac{3x}{8} + \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$$

$$382 \int \frac{dx}{\cos^2 ax} = \frac{1}{a} \operatorname{tg} ax$$

$$383 \int \frac{dx}{\cos^3 ax} = \frac{\sin ax}{2a \cos^2 ax} + \frac{1}{2a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$384 \int \cos px \cos qx \, dx = \frac{\sin(p-q)x}{2(p-q)} + \frac{\sin(p+q)x}{2(p+q)} \quad \text{Si } p = \pm q, \text{ véase 378}$$

$$385 \int \frac{dx}{1 - \cos ax} = \frac{1}{a} \cotg \frac{ax}{2}$$

$$386 \int \frac{x \, dx}{1 - \cos ax} = -\frac{x}{a} \cotg \frac{ax}{2} + \frac{2}{a^2} \ln \sin \frac{ax}{2}$$

$$387 \int \frac{dx}{1 + \cos ax} = \frac{1}{a} \operatorname{tg} \frac{ax}{2}$$

$$388 \int \frac{x \, dx}{1 + \cos ax} = \frac{x}{a} \operatorname{tg} \frac{ax}{2} + \frac{2}{a^2} \ln \cos \frac{ax}{2}$$

$$389 \int \frac{dx}{(1 - \cos ax)^2} = -\frac{1}{2a} \cotg \frac{ax}{2} - \frac{1}{6a} \cotg^3 \frac{ax}{2}$$

$$390 \int \frac{dx}{(1 + \cos ax)^2} = \frac{1}{2a} \operatorname{tg} \frac{ax}{2} + \frac{1}{6a} \operatorname{tg}^3 \frac{ax}{2}$$

$$391 \int \frac{dx}{p + q \cos ax} = \begin{cases} \frac{2}{a \sqrt{p^2 - q^2}} \operatorname{tg}^{-1} \sqrt{\frac{p-q}{p+q}} \cdot \operatorname{tg} \frac{ax}{2} & \text{Si } p = \pm q, \\ \frac{1}{a \sqrt{q^2 - p^2}} \ln \left( \frac{\operatorname{tg} \frac{ax}{2} + \sqrt{\frac{q+p}{q-p}}}{\operatorname{tg} \frac{ax}{2} - \sqrt{\frac{q+p}{q-p}}} \right) & \text{véanse 385 y 387} \end{cases}$$

$$392 \int \frac{dx}{(p + q \cos ax)^2} = \frac{\sin ax}{a(q^2 - p^2)(p + q \cos ax)} - \frac{p}{(q^2 - p^2)} \int \frac{dx}{p + q \cos ax}$$

Si  $p = \pm q$ , véanse 389 y 390

$$393 \int \frac{dx}{p^2 + q^2 \cos^2 ax} = \frac{1}{a p \sqrt{p^2 + q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tg} ax}{\sqrt{p^2 + q^2}}$$

$$394 \int \frac{dx}{p^2 - q^2 \cos^2 ax} = \begin{cases} \frac{1}{a p \sqrt{p^2 - q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tg} ax}{\sqrt{p^2 - q^2}} \\ \frac{1}{2 a p \sqrt{q^2 - p^2}} \ln \left( \frac{p \operatorname{tg} ax - \sqrt{q^2 - p^2}}{p \operatorname{tg} ax + \sqrt{q^2 - p^2}} \right) \end{cases}$$

$$395 \int x^m \cos ax \, dx = \frac{x^m \sin ax}{a} + \frac{m x^{m-1} \cos ax}{a^2} - \frac{m(m-1)}{a^2} \int x^{m-2} \cos ax \, dx$$

$$396 \int \frac{\cos ax}{x^n} \, dx = -\frac{\cos ax}{(n-1)x^{n-1}} - \frac{a}{n-1} \int \frac{\sin ax}{x^{n-1}} \, dx \quad \text{Véase 366}$$

$$397 \int \cos^n ax \, dx = \frac{\cos^{n-1} ax \sin ax}{a n} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$$

$$398 \int \frac{dx}{\cos^n ax} = \frac{\sin ax}{a(n-1)\cos^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\cos^{n-2} ax}$$

$$399 \int \frac{x \, dx}{\cos^n ax} = \frac{x \sin ax}{a(n-1)\cos^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\cos^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x \, dx}{\cos^{n-2} ax}$$

#### INTEGRALES CON $\sin ax$ y $\cos ax$

$$400 \int \sin ax \cos ax \, dx = \frac{\sin^2 ax}{2a}$$

$$401 \int \sin px \cos qx \, dx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)x}{2(p+q)}$$

$$402 \int \sin^n ax \cos ax \, dx = \frac{\sin^{n+1} ax}{(n+1)a} \quad \text{Si } n = -1, \text{ véase 441}$$

$$403 \int \sin ax \cos^n ax \, dx = -\frac{\cos^{n+1} ax}{(n+1)a} \quad \text{Si } n = -1, \text{ véase 430}$$

$$404 \int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

$$405 \int \frac{dx}{\sin ax \cos ax} = \frac{1}{a} \ln \operatorname{tg} ax$$

$$406 \int \frac{dx}{\sin^2 ax \cos ax} = \frac{1}{a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right) - \frac{1}{a \sin ax}$$

$$407 \int \frac{dx}{\sin ax \cos^2 ax} = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + \frac{1}{a \cos ax}$$

$$408 \int \frac{dx}{\sin^2 ax \cos^2 ax} = -\frac{2 \cotg 2ax}{a}$$

$$409 \int \frac{\sin^2 ax}{\cos ax} \, dx = \frac{1}{a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right) - \frac{\sin ax}{a}$$

$$410 \int \frac{\cos^2 ax}{\sin ax} \, dx = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + \frac{\cos ax}{a}$$

$$411 \int \frac{dx}{(1 \pm \sin ax) \cos ax} = \mp \frac{1}{2a(1 \pm \sin ax)} + \frac{1}{2a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$412 \int \frac{dx}{(1 \pm \cos ax) \sin ax} = \pm \frac{1}{2a(1 \pm \cos ax)} + \frac{1}{2a} \ln \operatorname{tg} \frac{ax}{2}$$

$$413 \int \frac{dx}{\sin ax \pm \cos ax} = \frac{1}{a \sqrt{2}} \ln \operatorname{tg} \left( \pm \frac{\pi}{8} + \frac{ax}{2} \right)$$

$$414 \int \frac{\sin ax \, dx}{\sin ax \pm \cos ax} = \frac{x}{2} \mp \frac{1}{2a} \ln (\sin ax \pm \cos ax)$$

$$415 \int \frac{\cos ax \, dx}{\sin ax \pm \cos ax} = \pm \frac{x}{2} + \frac{1}{2a} \ln(\sin ax \pm \cos ax)$$

$$416 \int \frac{\sin ax \, dx}{p + q \cos ax} = -\frac{1}{a} \ln(p + q \cos ax)$$

$$417 \int \frac{\cos ax \, dx}{p + q \sin ax} = \frac{1}{a} \ln(p + q \sin ax)$$

$$418 \int \frac{\sin ax \, dx}{(p + q \cos ax)^n} = \frac{1}{a q (n-1) (p + q \cos ax)^{n-1}}$$

$$419 \int \frac{\cos ax \, dx}{(p + q \sin ax)^n} = \frac{-1}{a q (n-1) (p + q \sin ax)^{n-1}}$$

$$420 \int \frac{dx}{p \sin ax + q \cos ax} = \frac{1}{a \sqrt{p^2 + q^2}} \ln \operatorname{tg} \left( \frac{ax + \operatorname{tg}^{-1}(q/p)}{2} \right)$$

$$421 \int \frac{dx}{p \sin ax + q \cos ax + r} = \begin{cases} \frac{2}{a \sqrt{r^2 - p^2 - q^2}} \operatorname{tg}^{-1} \left( \frac{p + (r - q) \operatorname{tg}(ax/2)}{\sqrt{r^2 - p^2 - q^2}} \right) \\ \frac{1}{a \sqrt{p^2 + q^2 - r^2}} \ln \left( \frac{p - \sqrt{p^2 + q^2 - r^2} + (r - q) \operatorname{tg}(ax/2)}{p + \sqrt{p^2 + q^2 - r^2} + (r - q) \operatorname{tg}(ax/2)} \right) \end{cases}$$

Si  $r = q$  véase 422. Si  $r^2 = p^2 + q^2$  véase 423.

$$422 \int \frac{dx}{p \sin ax + q(1 + \cos ax)} = \frac{1}{a p} \ln \left( q + p \operatorname{tg} \frac{ax}{2} \right)$$

$$423 \int \frac{dx}{p \sin ax + q \cos ax \pm \sqrt{p^2 + q^2}} = \frac{-1}{a \sqrt{p^2 + q^2}} \operatorname{tg} \left( \frac{\pi}{4} \mp \frac{ax + \operatorname{tg}^{-1}(q/p)}{2} \right)$$

$$424 \int \frac{dx}{p^2 \sin^2 ax + q^2 \cos^2 ax} = \frac{1}{a p q} \operatorname{tg}^{-1} \left( \frac{p \operatorname{tg} ax}{q} \right)$$

$$425 \int \frac{dx}{p^2 \sin^2 ax - q^2 \cos^2 ax} = \frac{1}{2 a p q} \ln \left( \frac{p \operatorname{tg} ax - q}{p \operatorname{tg} ax + q} \right)$$

$$426 \int \sin^m ax \cos^n ax \, dx = \begin{cases} -\frac{\sin^{m-1} ax \cos^{n+1} ax}{a(m+n)} + \frac{(m-1)}{(m+n)} \int \sin^{m-2} ax \cos^n ax \, dx \\ \frac{\sin^{m+1} ax \cos^{n-1} ax}{a(m+n)} + \frac{(n-1)}{(m+n)} \int \sin^m ax \cos^{n-2} ax \, dx \end{cases}$$

$$427 \int \frac{\sin^m ax}{\cos^n ax} \, dx = \begin{cases} \frac{\sin^{m-1} ax}{a(n-1) \cos^{n-1} ax} - \frac{m-1}{n-1} \int \frac{\sin^{m-2} ax}{\cos^{n-2} ax} \, dx \\ \frac{\sin^{m+1} ax}{a(n-1) \cos^{n-1} ax} - \frac{m-n+2}{n-1} \int \frac{\sin^m ax}{\cos^{n-2} ax} \, dx \\ -\frac{\sin^{m-1} ax}{a(m-n) \cos^{n-1} ax} + \frac{m-1}{m-n} \int \frac{\sin^{m-2} ax}{\cos^n ax} \, dx \end{cases}$$

$$428 \int \frac{\cos^m ax}{\sin^n ax} \, dx = \begin{cases} -\frac{\cos^{m-1} ax}{a(n-1) \sin^{n-1} ax} - \frac{m-1}{n-1} \int \frac{\cos^{m-2} ax}{\sin^{n-2} ax} \, dx \\ \frac{-\cos^{m+1} ax}{a(n-1) \sin^{n-1} ax} - \frac{m-n+2}{n-1} \int \frac{\cos^m ax}{\sin^{n-2} ax} \, dx \\ \frac{\cos^{m-1} ax}{a(m-n) \sin^{n-1} ax} + \frac{m-1}{m-n} \int \frac{\cos^{m-2} ax}{\sin^n ax} \, dx \end{cases}$$

$$429 \int \sin^m ax \cos^n ax \, dx = \begin{cases} \frac{1}{a(n-1) \sin^{n-1} ax \cos^{n-1} ax} + \frac{m+n-2}{n-1} \int \frac{dx}{\sin^m ax \cos^{n-2} ax} \\ \frac{-1}{a(m-1) \sin^{n-1} ax \cos^{n-1} ax} + \frac{m+n-2}{m-1} \int \frac{dx}{\sin^{m-2} ax \cos^n ax} \end{cases}$$

#### INTEGRALES CON $\operatorname{tg} ax$

$$430 \int \operatorname{tg} ax \, dx = -\frac{1}{a} \ln \cos ax = \frac{1}{a} \ln \sec ax$$

$$431 \int \operatorname{tg}^2 ax \, dx = \frac{\operatorname{tg} ax}{a} - x$$

$$432 \int \operatorname{tg}^3 ax \, dx = \frac{\operatorname{tg}^2 ax}{2a} + \frac{1}{a} \ln \cos ax$$

$$433 \int \operatorname{tg}^n ax \sec^2 ax \, dx = \frac{\operatorname{tg}^{n+1} ax}{(n+1)a}$$

$$434 \int \frac{\sec^2 ax}{\operatorname{tg} ax} \, dx = \frac{1}{a} \ln \operatorname{tg} ax$$

$$435 \int \frac{dx}{\operatorname{tg} ax} = \frac{1}{a} \ln \sin ax$$

$$436 \int x \operatorname{tg} ax \, dx = \frac{1}{a^2} \left( \frac{(ax)^3}{3} + \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} + \dots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \dots \right)$$

$B_n$  es  $n^\circ$  de Bernoulli tanto en 436 como en 437

$$437 \int \frac{\operatorname{tg} ax}{x} \, dx = ax + \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} + \dots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \dots$$

$$438 \int x \operatorname{tg}^2 ax \, dx = \frac{x \operatorname{tg} ax}{a} + \frac{1}{a^2} \ln \cos ax - \frac{x^2}{2}$$

$$439 \int \frac{dx}{p + q \operatorname{tg} ax} = \frac{px}{p^2 + q^2} + \frac{q}{a(p^2 + q^2)} \ln(q \sin ax + p \cos ax)$$

$$440 \int \operatorname{tg}^n ax \, dx = \frac{\operatorname{tg}^{n-1} ax}{(n-1)a} - \int \operatorname{tg}^{n-2} ax \, dx$$

# INTEGRALES CON $\cotg ax$

$$441 \int \cotg ax \, dx = -\frac{1}{a} \ln \sen ax$$

$$442 \int \cotg^2 ax \, dx = -\frac{\cotg ax}{a} - x$$

$$443 \int \cotg^3 ax \, dx = -\frac{\cotg^2 ax}{2a} - \frac{1}{a} \ln \sen ax$$

$$444 \int \cotg^n ax \operatorname{cosec}^2 ax \, dx = -\frac{\cotg^{n+1} ax}{(n+1)a}$$

$$445 \int \frac{\operatorname{cosec}^2 ax}{\cotg ax} \, dx = -\frac{1}{a} \ln \cotg ax$$

$$446 \int \frac{dx}{\cotg ax} = -\frac{1}{a} \ln \cos ax$$

$$447 \int x \cotg ax \, dx = \frac{1}{a^2} \left( ax - \frac{(ax)^3}{9} - \frac{(ax)^5}{225} - \dots - \frac{2^{2n} B_n (ax)^{2n+1}}{(2n+1)!} - \dots \right)$$

$B_n$  es n° de Bernoulli tanto en 447 como en 448

$$448 \int \frac{\cotg ax}{x} \, dx = -\frac{1}{ax} - \frac{ax}{3} - \frac{(ax)^3}{135} - \dots - \frac{2^{2n} B_n (ax)^{2n-1}}{(2n-1)(2n)!} - \dots$$

$$449 \int x \cotg^2 ax \, dx = \frac{x \cotg ax}{a} + \frac{1}{a^2} \ln \sen ax - \frac{x^2}{2}$$

$$450 \int \frac{dx}{p+q \cotg ax} = \frac{px}{p^2+q^2} - \frac{q}{a(p^2+q^2)} \ln(q \sen ax + p \cos ax)$$

$$451 \int \cotg^n ax \, dx = -\frac{\cotg^{n-1} ax}{(n-1)a} - \int \cotg^{n-2} ax \, dx$$

# INTEGRALES CON $\sec ax$

$$452 \int \sec ax \, dx = \frac{1}{a} \ln (\sec ax + \tg ax) = \frac{1}{a} \ln \tg \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$453 \int \sec^2 ax \, dx = \frac{\tg ax}{a}$$

$$454 \int \sec^3 ax \, dx = \frac{\sec ax \tg ax}{2a} + \frac{1}{2a} \ln (\sec ax + \tg ax)$$

$$455 \int \sec^n ax \tg ax \, dx = \frac{\sec^n ax}{n a}$$

$$456 \int \frac{dx}{\sec ax} = \frac{\sen ax}{a}$$

$$457 \int x \sec ax \, dx = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \dots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\} \quad E_n \text{ es n° de Euler}$$

$$458 \int \frac{\sec ax}{x} \, dx = \ln x + \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} + \frac{61(ax)^6}{4320} + \dots + \frac{E_n (ax)^{2n}}{2n(2n)!} + \dots$$

$$459 \int x \sec^2 ax \, dx = \frac{x}{a} \tg ax + \frac{1}{a^2} \ln \cos ax$$

$$460 \int \frac{dx}{q+p \sec ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{q+p \cos ax} \quad \text{Véase 391}$$

$$461 \int \sec^n ax \, dx = \frac{\sec^{n-2} ax \tg ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx$$

# INTEGRALES CON $\operatorname{cosec} ax$

$$462 \int \operatorname{cosec} ax \, dx = \frac{1}{a} \ln (\operatorname{cosec} ax - \cotg ax) = \frac{1}{a} \ln \tg \frac{ax}{2}$$

$$463 \int \operatorname{cosec}^2 ax \, dx = -\frac{\cotg ax}{a}$$

$$464 \int \operatorname{cosec}^3 ax \, dx = -\frac{\operatorname{cosec} ax \cotg ax}{2a} + \frac{1}{2a} \ln \tg \frac{ax}{2}$$

$$465 \int \operatorname{cosec}^n ax \cotg ax \, dx = -\frac{\operatorname{cosec}^n ax}{n a}$$

$$466 \int \frac{dx}{\operatorname{cosec} x} = -\frac{\cos ax}{a}$$

$$467 \int x \operatorname{cosec} ax \, dx = \frac{1}{a^2} \left\{ ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \dots + \frac{2(2^{2n-1}-1)B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}$$

$B_n$  es n° de Bernoulli

$$468 \int \frac{\operatorname{cosec} ax}{x} \, dx = -\frac{1}{ax} + \frac{ax}{6} + \frac{7(ax)^3}{1080} + \dots + \frac{2(2^{2n-1}-1)B_n (ax)^{2n-1}}{(2n-1)(2n)!} + \dots$$

$$469 \int x \operatorname{cosec}^2 ax \, dx = -\frac{x}{a} \cotg ax + \frac{1}{a^2} \ln \sen ax$$

$$470 \int \frac{dx}{q+p \operatorname{cosec} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{q+p \sen ax} \quad \text{Véase 361}$$

$$471 \int \operatorname{cosec}^n ax \, dx = -\frac{\operatorname{cosec}^{n-2} ax \cotg ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} ax \, dx$$

# INTEGRALES DE FUNCIONES TRIGONOMETRICAS INVERSAS

$$472 \int \sen^{-1} \frac{x}{a} \, dx = x \sen^{-1} \frac{x}{a} + \sqrt{a^2 - x^2}$$

$$473 \int x \sen^{-1} \frac{x}{a} \, dx = \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \sen^{-1} \frac{x}{a} + \frac{x \sqrt{a^2 - x^2}}{4}$$

$$474 \int x^2 \sen^{-1} \frac{x}{a} \, dx = \frac{x^3}{3} \sen^{-1} \frac{x}{a} + \frac{(x^2 + 2a^2) \sqrt{a^2 - x^2}}{9}$$

$$\begin{aligned}
475 \quad & \int x^m \operatorname{sech}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{sech}^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{a^2 - x^2}} dx \\
476 \quad & \int \frac{\operatorname{sech}^{-1} \frac{x}{a}}{x} dx = \frac{x}{a} + \frac{(x/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 (x/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 (x/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots \\
477 \quad & \int \frac{\operatorname{sech}^{-1} \frac{x}{a}}{x^2} dx = -\frac{\operatorname{sech}^{-1} \frac{x}{a}}{x} - \frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) \\
478 \quad & \int \left( \operatorname{sech}^{-1} \frac{x}{a} \right)^2 dx = x \left( \operatorname{sech}^{-1} \frac{x}{a} \right)^2 - 2x + 2 \sqrt{a^2 - x^2} \operatorname{sech}^{-1} \frac{x}{a} \\
479 \quad & \int \cos^{-1} \frac{x}{a} dx = x \cos^{-1} \frac{x}{a} - \sqrt{a^2 - x^2} \\
480 \quad & \int x \cos^{-1} \frac{x}{a} dx = \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \cos^{-1} \frac{x}{a} - \frac{x \sqrt{a^2 - x^2}}{4} \\
481 \quad & \int x^2 \cos^{-1} \frac{x}{a} dx = \frac{x^3}{3} \cos^{-1} \frac{x}{a} - \frac{(x^2 + 2a^2) \sqrt{a^2 - x^2}}{9} \\
482 \quad & \int x^m \cos^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \cos^{-1} \frac{x}{a} + \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{a^2 - x^2}} dx \\
483 \quad & \int \frac{\cos^{-1} \frac{x}{a}}{x} dx = \frac{\pi}{2} \ln x - \int \frac{\operatorname{sech}^{-1} \frac{x}{a}}{x} dx \quad \text{Véase 476} \\
484 \quad & \int \frac{\cos^{-1} \frac{x}{a}}{x^2} dx = -\frac{\cos^{-1} (x/a)}{x} + \frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) \\
485 \quad & \int \left( \cos^{-1} \frac{x}{a} \right)^2 dx = x \left( \cos^{-1} \frac{x}{a} \right)^2 - 2x - 2 \sqrt{a^2 - x^2} \cos^{-1} \frac{x}{a} \\
486 \quad & \int \operatorname{tg}^{-1} \frac{x}{a} dx = x \operatorname{tg}^{-1} \frac{x}{a} - \frac{a}{2} \ln (x^2 + a^2) \\
487 \quad & \int x \operatorname{tg}^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 + a^2) \operatorname{tg}^{-1} \frac{x}{a} - \frac{ax}{2} \\
488 \quad & \int x^2 \operatorname{tg}^{-1} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{tg}^{-1} \frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6} \ln (x^2 + a^2) \\
489 \quad & \int x^m \operatorname{tg}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{tg}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{x^2 + a^2} dx \\
490 \quad & \int \frac{\operatorname{tg}^{-1} (x/a)}{x} dx = \frac{x}{a} - \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} - \frac{(x/a)^7}{7^2} + \dots \\
491 \quad & \int \frac{\operatorname{tg}^{-1} (x/a)}{x^2} dx = -\frac{1}{x} \operatorname{tg}^{-1} \frac{x}{a} - \frac{1}{2a} \ln \left( \frac{x^2 + a^2}{x^2} \right) \\
492 \quad & \int \cotg^{-1} \frac{x}{a} dx = x \cotg^{-1} \frac{x}{a} + \frac{a}{2} \ln (x^2 + a^2) \\
493 \quad & \int x \cotg^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 + a^2) \cotg^{-1} \frac{x}{a} + \frac{ax}{2} \\
494 \quad & \int x^2 \cotg^{-1} \frac{x}{a} dx = \frac{x^3}{3} \cotg^{-1} \frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6} \ln (x^2 + a^2)
\end{aligned}$$

$$495 \quad \int x^m \cotg^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \cotg^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^{m+1}}{x^2 + a^2} dx$$

$$496 \quad \int \frac{\cotg^{-1} (x/a)}{x} dx = \frac{\pi}{2} \ln x - \int \frac{\operatorname{tg}^{-1} \frac{x}{a}}{x} dx \quad \text{Véase 490}$$

$$497 \quad \int \frac{\cotg^{-1} (x/a)}{x^2} dx = -\frac{1}{x} \cotg^{-1} \frac{x}{a} + \frac{1}{2a} \ln \left( \frac{x^2 + a^2}{x^2} \right)$$

$$498 \quad \int \sec^{-1} \frac{x}{a} dx = \begin{cases} x \sec^{-1} \frac{x}{a} - a \ln (x + \sqrt{x^2 - a^2}); & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ x \sec^{-1} \frac{x}{a} + a \ln (x + \sqrt{x^2 - a^2}); & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$499 \quad \int x \sec^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^2}{2} \sec^{-1} \frac{x}{a} - \frac{a \sqrt{x^2 - a^2}}{2}; & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^2}{2} \sec^{-1} \frac{x}{a} + \frac{a \sqrt{x^2 - a^2}}{2}; & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$500 \quad \int x^2 \sec^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^3}{3} \sec^{-1} \frac{x}{a} - \frac{a x \sqrt{x^2 - a^2}}{2} - \frac{a^3}{6} \ln (x + \sqrt{x^2 - a^2}); & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^3}{3} \sec^{-1} \frac{x}{a} + \frac{a x \sqrt{x^2 - a^2}}{2} + \frac{a^3}{6} \ln (x + \sqrt{x^2 - a^2}); & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$501 \quad \int x^m \sec^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \sec^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2 - a^2}} dx; & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^{m+1}}{m+1} \sec^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2 - a^2}} dx; & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$502 \quad \int \frac{\sec^{-1} (x/a)}{x} dx = \frac{\pi}{2} \ln x + \frac{a}{x} + \frac{(a/x)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 (a/x)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 (a/x)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots$$

$$503 \quad \int \frac{\sec^{-1} (x/a)}{x^2} dx = \begin{cases} -\frac{\sec^{-1} (x/a)}{x} + \frac{\sqrt{x^2 - a^2}}{a x}; & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ -\frac{\sec^{-1} (x/a)}{x} - \frac{\sqrt{x^2 - a^2}}{a x}; & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$504 \quad \int \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} x \operatorname{cosec}^{-1} \frac{x}{a} + a \ln (x + \sqrt{x^2 - a^2}); & 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ x \operatorname{cosec}^{-1} \frac{x}{a} - a \ln (x + \sqrt{x^2 - a^2}); & -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$505 \quad \int x \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^2}{2} \operatorname{cosec}^{-1} \frac{x}{a} + \frac{a \sqrt{x^2 - a^2}}{2}; & 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^2}{2} \operatorname{cosec}^{-1} \frac{x}{a} - \frac{a \sqrt{x^2 - a^2}}{2}; & -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

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$$506 \int x^2 \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x}{3} \operatorname{cosec}^{-1} \frac{x}{a} + \frac{ax \sqrt{x^2 - a^2}}{2} + \frac{a}{6} \ln(x + \sqrt{x^2 - a^2}); 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^3}{3} \operatorname{cosec}^{-1} \frac{x}{a} - \frac{ax \sqrt{x^2 - a^2}}{2} - \frac{a^3}{6} \ln(x + \sqrt{x^2 - a^2}); -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$507 \int x^m \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \operatorname{cosec}^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 - a^2}}; 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^{m+1}}{m+1} \operatorname{cosec}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 - a^2}}; \frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < \pi \end{cases}$$

$$508 \int \frac{\operatorname{cosec}^{-1}(x/a)}{x} dx = - \left( \frac{a}{x} + \frac{(a/x)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 (a/x)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 (a/x)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots \right)$$

$$509 \int \frac{\operatorname{cosec}^{-1}(x/a)}{x^2} dx = \begin{cases} -\frac{\operatorname{cosec}^{-1}(x/a)}{x} - \frac{\sqrt{x^2 - a^2}}{a x}; 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ -\frac{\operatorname{cosec}^{-1}(x/a)}{x} + \frac{\sqrt{x^2 - a^2}}{a x}; -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

#### INTEGRALES CON $e^{ax}$

$$510 \int e^{ax} dx = \frac{e^{ax}}{a}$$

$$511 \int x e^{ax} dx = \frac{e^{ax}}{a} \left( x - \frac{1}{a} \right)$$

$$512 \int x^2 e^{ax} dx = \frac{e^{ax}}{a} \left( x^2 - \frac{2x}{a} + \frac{2}{a^2} \right)$$

$$513 \int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \left( = \sum_{k=1}^n \frac{(-1)^k k!}{a^k} \text{ Si } n \text{ es natural} \right)$$

$$514 \int \frac{e^{ax}}{x} dx = \ln x + \frac{ax}{1 \cdot 1!} + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^3}{3 \cdot 3!} + \dots$$

$$515 \int \frac{e^{ax}}{x^n} dx = \frac{-e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{e^{ax}}{x^{n-1}} dx$$

$$516 \int \frac{dx}{p + q e^{ax}} = \frac{x}{p} - \frac{1}{ap} \ln(p + q e^{ax})$$

$$517 \int \frac{dx}{(p + q e^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p + q e^{ax})} - \frac{1}{ap^2} \ln(p + q e^{ax})$$

$$518 \int \frac{dx}{p e^{ax} + q e^{-ax}} = \begin{cases} \frac{1}{a \sqrt{pq}} \operatorname{tg}^{-1} \left( \sqrt{\frac{p}{q}} e^{ax} \right) \\ \frac{1}{2a \sqrt{-pq}} \ln \left( \frac{e^{ax} - \sqrt{-q/p}}{e^{ax} + \sqrt{-q/p}} \right) \end{cases}$$

$$519 \int e^{ax} \operatorname{sen} bx dx = \frac{e^{ax} (a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2} \quad \text{es.}$$

$$520 \int e^{ax} \cos bx dx = \frac{e^{ax} (a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2}$$

$$521 \int x e^{ax} \operatorname{sen} bx dx = \frac{x e^{ax} (a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2} - \frac{e^{ax} ((a^2 - b^2) \operatorname{sen} bx - 2ab \cos bx)}{(a^2 + b^2)^2}$$

$$522 \int x e^{ax} \cos bx dx = \frac{x e^{ax} (a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2} - \frac{e^{ax} ((a^2 - b^2) \cos bx + 2ab \operatorname{sen} bx)}{(a^2 + b^2)^2}$$

$$523 \int e^{ax} \ln x dx = \frac{e^{ax} \ln x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$$

$$524 \int e^{ax} \operatorname{sen}^n bx dx = \frac{e^{ax} \operatorname{sen}^{n-1} bx (a \operatorname{sen} bx - nb \cos bx)}{a^2 + n^2 b^2} + \frac{n(n-1)b^2}{a^2 + n^2 b^2} \int e^{ax} \operatorname{sen}^{n-2} bx dx$$

$$525 \int e^{ax} \cos^n bx dx = \frac{e^{ax} \cos^{n-1} bx (a \cos bx + nb \operatorname{sen} bx)}{a^2 + n^2 b^2} + \frac{n(n-1)b^2}{a^2 + n^2 b^2} \int e^{ax} \cos^{n-2} bx dx$$

#### INTEGRALES CON $\ln x$

$$526 \int \ln x dx = x \ln x - x$$

$$527 \int x \ln x dx = \frac{x^2}{2} \left( \ln x - \frac{1}{2} \right)$$

$$528 \int x^m \ln x dx = \frac{x^{m+1}}{m+1} \left( \ln x - \frac{1}{m+1} \right) \quad \text{Si } m = -1 \text{ véase 529}$$

$$529 \int \frac{\ln x}{x} dx = \frac{1}{2} \ln^2 x$$

$$530 \int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x}$$

$$531 \int \ln^2 x dx = x \ln^2 x - 2x \ln x + 2x$$

$$532 \int \frac{\ln^n x}{x} dx = \frac{\ln^{n+1} x}{n+1} \quad \text{Si } n = -1 \text{ véase 533}$$

$$533 \int \frac{dx}{x \ln x} = \ln(\ln x)$$

$$534 \int \frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{\ln^2 x}{2 \cdot 2!} + \frac{\ln^3 x}{3 \cdot 3!} + \dots$$

$$535 \int \frac{x^m dx}{\ln x} = \ln(\ln x) + (m+1) \ln x + \frac{(m+1)^2 \ln^2 x}{2 \cdot 2!} + \frac{(m+1)^3 \ln^3 x}{3 \cdot 3!} + \dots$$

$$536 \int \ln^n x dx = x \ln^n x - n \int \ln^{n-1} x dx$$

$$537 \int x^m \ln^n x dx = \frac{\ln^n x}{m+1} - \frac{n}{m+1} \int x^m \ln^{n-1} x dx \quad \text{Si } m = -1 \text{ véase 532}$$

$$538 \int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) - 2x + 2a \operatorname{tg}^{-1} \frac{x}{a}$$

$$539 \int \ln(x^2 - a^2) dx = x \ln(x^2 - a^2) - 2x + a \ln \left( \frac{x+a}{x-a} \right)$$

$$540 \int x^m \ln(x^2 \pm a^2) dx = \frac{x^{m+1} \ln(x^2 \pm a^2)}{m+1} - \frac{2}{m+1} \int \frac{x^{m+2}}{(x^2 \pm a^2)} dx$$

**INTEGRALES CON  $\sinh ax$**

$$\begin{aligned}
 541 \quad & \int \sinh ax \, dx = \frac{\cosh ax}{a} \\
 542 \quad & \int x \sinh ax \, dx = -\frac{\sinh ax}{a^2} + \frac{x \cosh ax}{a} \\
 543 \quad & \int x^2 \sinh ax \, dx = -\frac{2x}{a^2} \sinh ax + \left( \frac{2}{a^3} + \frac{x^2}{a} \right) \cosh ax \\
 544 \quad & \int \frac{\sinh ax}{x} \, dx = ax + \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \dots = \sum_{n=1}^{\infty} \frac{(ax)^{2n-1}}{(2n-1)(2n-1)!} \\
 545 \quad & \int \frac{\sinh ax}{x^2} \, dx = -\frac{\sinh ax}{x} + a \int \frac{\cosh ax}{x} \, dx \quad \text{Véase 566} \\
 546 \quad & \frac{dx}{\sinh ax} = \frac{1}{a} \ln \tanh \frac{ax}{2} \\
 547 \quad & \int \frac{x \, dx}{\sinh ax} = \frac{1}{a^2} \left\{ ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \dots + \frac{2(-1)^n (2^{2n-1} - 1) B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}
 \end{aligned}$$

$B_n$  es nº de Bernoulli

$$\begin{aligned}
 548 \quad & \int \sinh^2 ax \, dx = -\frac{x}{2} + \frac{\sinh ax \cosh ax}{2a} \\
 549 \quad & \int x \sinh^2 ax \, dx = -\frac{x^2}{4} + \frac{x \sinh 2ax}{4a} - \frac{\cosh 2ax}{8a^2} \\
 550 \quad & \int \frac{dx}{\sinh^2 ax} = -\frac{1}{a} \coth ax \\
 551 \quad & \int \sinh px \sinh qx \, dx = -\frac{\sinh(p-q)x}{2(p-q)} + \frac{\sinh(p+q)x}{2(p+q)} \quad \text{Si } p = \pm q, \text{ véase 548} \\
 552 \quad & \int \sinh px \cosh qx \, dx = \frac{p \cosh px \sinh qx - q \sinh px \cosh qx}{p^2 + q^2} \\
 553 \quad & \int \sinh px \cos qx \, dx = \frac{p \cosh px \cos qx + q \sinh px \sin qx}{p^2 + q^2} \\
 554 \quad & \int \frac{dx}{p + q \sinh ax} = \frac{1}{a \sqrt{p^2 + q^2}} \ln \left( \frac{q e^{ax} + p - \sqrt{p^2 + q^2}}{q e^{ax} + p + \sqrt{p^2 + q^2}} \right) \\
 555 \quad & \int \frac{dx}{(p + q \sinh ax)^2} = \frac{-q \cosh ax}{a(p^2 + q^2)(p + q \sinh ax)} + \frac{p}{(p^2 + q^2)} \int \frac{dx}{p + q \sinh ax}
 \end{aligned}$$

$$556 \quad \int \frac{dx}{p^2 + q^2 \sinh^2 ax} = \begin{cases} \frac{1}{a p \sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \frac{\sqrt{q^2 - p^2} \cdot \tanh ax}{p} \\ \frac{1}{2 a p \sqrt{p^2 - q^2}} \ln \left( \frac{p + \sqrt{p^2 - q^2} \cdot \tanh ax}{p - \sqrt{p^2 - q^2} \cdot \tanh ax} \right) \end{cases}$$

$$557 \quad \int \frac{dx}{p^2 - q^2 \sinh^2 ax} = \frac{1}{2 a p \sqrt{p^2 + q^2}} \ln \left( \frac{p + \sqrt{p^2 + q^2} \cdot \tanh ax}{p - \sqrt{p^2 + q^2} \cdot \tanh ax} \right)$$

$$558 \quad \int x^n \sinh ax \, dx = \frac{x^n \cosh ax}{a} - \frac{n}{a} \int x^{n-1} \cosh ax \, dx \quad \text{Véase 586}$$

$$559 \quad \int \frac{\sinh ax}{x^n} \, dx = -\frac{\sinh ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\cosh ax}{x^{n-1}} \, dx \quad \text{Véase 587}$$

$$560 \quad \int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{a n} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx$$

$$561 \quad \int \frac{dx}{\sinh^n ax} = \frac{-\cosh ax}{a(n-1)\sinh^{n-1} ax} - \frac{n-2}{n-1} \int \frac{dx}{\sinh^{n-2} ax}$$

$$562 \quad \int \frac{x \, dx}{\sinh^n ax} = \frac{-x \cosh ax}{a(n-1)\sinh^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\sinh^{n-2} ax} - \frac{n-2}{n-1} \int \frac{x \, dx}{\sinh^{n-2} ax}$$

**INTEGRALES CON  $\cosh ax$**

$$\begin{aligned}
 563 \quad & \int \cosh ax \, dx = \frac{\sinh ax}{a} \\
 564 \quad & \int x \cosh ax \, dx = -\frac{\cosh ax}{a^2} + \frac{x \sinh ax}{a} \\
 565 \quad & \int x^2 \cosh ax \, dx = -\frac{2x}{a^2} \cosh ax + \left( \frac{x^2}{a} + \frac{2}{a^3} \right) \sinh ax \\
 566 \quad & \int \frac{\cosh ax}{x} \, dx = \ln x + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} + \frac{(ax)^6}{6 \cdot 6!} + \dots = \ln x + \sum_{n=1}^{\infty} \frac{(ax)^{2n}}{(2n) \cdot (2n)!} \\
 567 \quad & \int \frac{\cosh ax}{x^2} \, dx = -\frac{\cosh ax}{x} + a \int \frac{\sinh ax}{x} \, dx \quad \text{Véase 544} \\
 568 \quad & \int \frac{dx}{\cosh ax} = \frac{2}{a} \operatorname{tg}^{-1} e^{ax} \\
 569 \quad & \int \frac{x \, dx}{\cosh ax} = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} - \dots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\} \quad E_n \text{ es nº de Euler} \\
 570 \quad & \int \cosh^2 ax \, dx = \frac{x}{2} + \frac{\sinh ax \cosh ax}{2} \\
 571 \quad & \int x \cosh^2 ax \, dx = \frac{x^2}{4} + \frac{x \sinh 2ax}{4a} - \frac{\cosh 2ax}{8a^2} \\
 572 \quad & \int \frac{dx}{\cosh^2 ax} = \frac{1}{a} \tanh ax \\
 573 \quad & \int \cosh px \cosh qx \, dx = \frac{\sinh(p-q)x}{2(p-q)} + \frac{\sinh(p+q)x}{2(p+q)} \quad \text{Si } p = \pm q, \text{ véase 570} \\
 574 \quad & \int \cosh px \sinh qx \, dx = \frac{p \sinh px \sinh qx - q \cosh px \cosh qx}{p^2 + q^2} \\
 575 \quad & \int \cosh px \cos qx \, dx = \frac{p \sinh px \cos qx + q \cosh px \sin qx}{p^2 + q^2}
 \end{aligned}$$

$$576 \int \frac{dx}{1 - \cosh ax} = \frac{1}{a} \operatorname{cotgh} \frac{ax}{2}$$

$$577 \int \frac{x dx}{1 - \cosh ax} = \frac{x}{a} \operatorname{cotgh} \frac{ax}{2} - \frac{2}{a^2} \ln \sinh \frac{ax}{2}$$

$$578 \int \frac{dx}{1 + \cosh ax} = \frac{1}{a} \operatorname{tgh} \frac{ax}{2}$$

$$579 \int \frac{x dx}{1 + \cosh ax} = \frac{x}{a} \operatorname{tgh} \frac{ax}{2} - \frac{2}{a^2} \ln \cosh \frac{ax}{2}$$

$$580 \int \frac{dx}{(1 - \cosh ax)^2} = \frac{1}{2a} \operatorname{cotgh} \frac{ax}{2} - \frac{1}{6a} \operatorname{cotgh}^3 \frac{ax}{2}$$

$$581 \int \frac{dx}{(1 + \cosh ax)^2} = \frac{1}{2a} \operatorname{tgh} \frac{ax}{2} - \frac{1}{6a} \operatorname{tgh}^3 \frac{ax}{2}$$

$$582 \int \frac{dx}{p + q \cosh ax} = \begin{cases} \frac{2}{a \sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \frac{p + q e^{ax}}{\sqrt{q^2 - p^2}} \\ \frac{1}{a \sqrt{p^2 - q^2}} \ln \left( \frac{q e^{ax} + p - \sqrt{p^2 - q^2}}{q e^{ax} + p + \sqrt{p^2 - q^2}} \right) \end{cases}$$

$$583 \int \frac{dx}{(p + q \cosh ax)^2} = \frac{q \sinh ax}{a (q^2 - p^2) (p + q \cosh ax)} - \frac{p}{(q^2 - p^2)} \int \frac{dx}{p + q \cosh ax}$$

$$584 \int \frac{dx}{p^2 + q^2 \cosh^2 ax} = \begin{cases} \frac{1}{2 a p \sqrt{p^2 + q^2}} \ln \left( \frac{p \operatorname{tgh} ax + \sqrt{p^2 + q^2}}{p \operatorname{tgh} ax - \sqrt{p^2 + q^2}} \right) \\ \frac{1}{a p \sqrt{p^2 + q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tgh} ax}{\sqrt{p^2 + q^2}} \end{cases}$$

$$585 \int \frac{dx}{p^2 - q^2 \cosh^2 ax} = \begin{cases} \frac{-1}{a p \sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tgh} ax}{\sqrt{q^2 - p^2}} \\ \frac{1}{2 a p \sqrt{p^2 - q^2}} \ln \left( \frac{p \operatorname{tgh} ax + \sqrt{p^2 - q^2}}{p \operatorname{tgh} ax - \sqrt{p^2 - q^2}} \right) \end{cases}$$

$$586 \int x^n \cosh ax dx = \frac{x^n \sinh ax}{a} - \frac{n}{a} \int x^{n-1} \sinh ax dx \quad \text{Véase 558}$$

$$587 \int \frac{\cosh ax}{x^n} dx = -\frac{\cosh ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\sinh ax}{x^{n-1}} dx \quad \text{Véase 559}$$

$$588 \int \cosh^n ax dx = \frac{\cosh^{n-1} ax \sinh ax}{a n} + \frac{n-1}{n} \int \cosh^{n-2} ax dx$$

$$589 \int \frac{dx}{\cosh^n ax} = \frac{\sinh ax}{a (n-1) \cosh^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cosh^{n-2} ax}$$

$$590 \int \frac{x dx}{\cosh^n ax} = \frac{x \sinh ax}{a (n-1) \cosh^{n-1} ax} + \frac{1}{a^2 (n-1) (n-2) \cosh^{n-2} ax} + \frac{n-2}{n-1} \int \frac{x dx}{\cosh^{n-2} ax}$$

#### INTEGRALES CON $\sinh ax$ y $\cosh ax$

$$591 \int \sinh ax \cosh ax dx = \frac{\sinh^2 ax}{2a}$$

$$592 \int \sinh px \cosh qx dx = \frac{\cosh (p-q)x}{2(p-q)} + \frac{\cosh (p+q)x}{2(p+q)}$$

$$593 \int \sinh^n ax \cosh ax dx = \frac{\sinh^{n+1} ax}{(n+1)a} \quad \text{Si } n = -1, \text{ véase 615}$$

$$594 \int \sinh ax \cosh^n ax dx = \frac{\cosh^{n+1} ax}{(n+1)a} \quad \text{Si } n = -1, \text{ véase 604}$$

$$595 \int \sinh^2 ax \cosh^2 ax dx = -\frac{x}{8} + \frac{\sinh 4ax}{32a}$$

$$596 \int \frac{dx}{\sinh ax \cosh ax} = \frac{1}{a} \ln \operatorname{tgh} ax$$

$$597 \int \frac{dx}{\sinh^2 ax \cosh ax} = -\frac{1}{a} \operatorname{tg}^{-1} \sinh ax - \frac{\cosh ax}{a}$$

$$598 \int \frac{dx}{\sinh ax \cosh^2 ax} = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\operatorname{sech} ax}{a}$$

$$599 \int \frac{dx}{\sinh^2 ax \cosh^2 ax} = -\frac{2 \operatorname{cotgh} 2ax}{a}$$

$$600 \int \frac{\sinh^2 ax}{\cosh ax} dx = -\frac{1}{a} \operatorname{tg}^{-1} \sinh ax + \frac{\sinh ax}{a}$$

$$601 \int \frac{\cosh^2 ax}{\sinh ax} dx = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\cosh ax}{a}$$

$$602 \int \frac{dx}{(1 + \sinh ax) \cosh ax} = \frac{1}{2a} \ln \left( \frac{1 + \sinh ax}{\cosh ax} \right) + \frac{1}{a} \operatorname{tg}^{-1} e^{ax}$$

$$603 \int \frac{dx}{(\cosh ax \pm 1) \sinh ax} = \pm \frac{1}{2a (\cosh ax \pm 1)} \pm \frac{1}{2a} \ln \operatorname{tgh} \frac{ax}{2}$$

#### INTEGRALES CON $\operatorname{tgh} ax$

$$604 \int \operatorname{tgh} ax dx = \frac{1}{a} \ln \cosh ax$$

$$605 \int \operatorname{tgh}^2 ax dx = -\frac{\operatorname{tgh} ax}{a} + x$$

$$606 \int \operatorname{tgh}^3 ax dx = -\frac{\operatorname{tgh}^2 ax}{2a} + \frac{1}{a} \ln \cosh ax$$



$$\begin{aligned}
607 \quad & \int \operatorname{tgh}^n ax \operatorname{sech}^2 ax \, dx = \frac{\operatorname{tgh}^{n+1} ax}{(n+1)a} \\
608 \quad & \int \frac{\operatorname{sech}^2 ax}{\operatorname{tgh} ax} \, dx = \frac{1}{a} \ln \operatorname{tgh} ax \\
609 \quad & \int \frac{dx}{\operatorname{tgh} ax} = \frac{1}{a} \ln \sinh ax \\
610 \quad & \int x \operatorname{tgh} ax \, dx = \frac{1}{a^2} \left\{ \frac{(ax)^3}{3} - \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} - \dots + \frac{(-1)^{n-1} 2^{2n} (2^{2n} - 1) B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}
\end{aligned}$$

$B_n$  es  $n^\circ$  de Bernoulli tanto en 610 como en 611

$$\begin{aligned}
611 \quad & \int \frac{\operatorname{tgh} ax}{x} \, dx = ax - \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} - \dots + \frac{(-1)^{n-1} 2^{2n} (2^{2n} - 1) B_n (ax)^{2n+1}}{(2n-1)(2n)!} + \dots \\
612 \quad & \int x \operatorname{tgh}^2 ax \, dx = -\frac{x \operatorname{tgh} ax}{a} + \frac{1}{a^2} \ln \cosh ax + \frac{x^2}{2} \\
613 \quad & \int \frac{dx}{p + q \operatorname{tgh} ax} = \frac{px}{p^2 - q^2} - \frac{q}{a(p^2 - q^2)} \ln(q \sinh ax + p \cosh ax) \\
614 \quad & \int \operatorname{tgh}^n ax \, dx = -\frac{\operatorname{tgh}^{n-1} ax}{(n-1)a} + \int \operatorname{tgh}^{n-2} ax \, dx
\end{aligned}$$

#### INTEGRALES CON $\operatorname{cotgh} ax$

$$\begin{aligned}
615 \quad & \int \operatorname{cotgh} ax \, dx = \frac{1}{a} \ln \sinh ax \\
616 \quad & \int \operatorname{cotgh}^2 ax \, dx = -\frac{\operatorname{cotgh} ax}{a} + x \\
617 \quad & \int \operatorname{cotgh}^3 ax \, dx = -\frac{\operatorname{cotgh}^2 ax}{2a} + \frac{1}{a} \ln \sinh ax \\
618 \quad & \int \operatorname{cotgh}^n ax \operatorname{cosech}^2 ax \, dx = -\frac{\operatorname{cotgh}^{n+1} ax}{(n+1)a} \\
619 \quad & \int \frac{\operatorname{cosech}^2 ax}{\operatorname{cotgh} ax} \, dx = -\frac{1}{a} \ln \operatorname{cotgh} ax \\
620 \quad & \int \frac{dx}{\operatorname{cotgh} ax} = \frac{1}{a} \ln \cosh ax \\
621 \quad & \int x \operatorname{cotgh} ax \, dx = \frac{1}{a^2} \left\{ ax + \frac{(ax)^3}{9} - \frac{(ax)^5}{225} + \dots + \frac{(-1)^{n-1} 2^{2n} B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}
\end{aligned}$$

$B_n$  es  $n^\circ$  de Bernoulli tanto en 621 como en 622

$$\begin{aligned}
622 \quad & \int \frac{\operatorname{cotgh} ax}{x} \, dx = -\frac{1}{ax} + \frac{ax}{3} - \frac{(ax)^3}{135} + \dots + \frac{(-1)^n 2^{2n} B_n (ax)^{2n+1}}{(2n-1)(2n)!} + \dots \\
623 \quad & \int x \operatorname{cotgh}^2 ax \, dx = -\frac{x \operatorname{cotgh} ax}{a} + \frac{1}{a^2} \ln \sinh ax + \frac{x^2}{2} \\
624 \quad & \int \frac{dx}{p + q \operatorname{cotgh} ax} = \frac{px}{p^2 - q^2} - \frac{q}{a(p^2 - q^2)} \ln(p \sinh ax + q \cosh ax)
\end{aligned}$$

$$625 \quad \int \operatorname{cotgh}^n ax \, dx = -\frac{\operatorname{cotgh}^{n-1} ax}{(n-1)a} + \int \operatorname{cotgh}^{n-2} ax \, dx$$

#### INTEGRALES CON $\operatorname{sech} ax$

$$\begin{aligned}
626 \quad & \int \operatorname{sech} ax \, dx = \frac{2}{a} \operatorname{tg}^{-1} e^{ax} \\
627 \quad & \int \operatorname{sech}^2 ax \, dx = \frac{\operatorname{tgh} ax}{a} \\
628 \quad & \int \operatorname{sech}^3 ax \, dx = \frac{\operatorname{sech} ax \operatorname{tgh} ax}{2a} + \frac{1}{2a} \operatorname{tg}^{-1} \sinh ax \\
629 \quad & \int \operatorname{sech}^n ax \operatorname{tgh} ax \, dx = -\frac{\operatorname{sech}^n ax}{n a} \\
630 \quad & \int \frac{dx}{\operatorname{sech} ax} = \frac{\sinh ax}{a} \\
631 \quad & \int x \operatorname{sech} ax \, dx = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} - \dots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\} \quad E_n \text{ es}
\end{aligned}$$

$n^\circ$  de Euler

$$\begin{aligned}
632 \quad & \int \frac{\operatorname{sech} ax}{x} \, dx = \ln x - \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} - \frac{61(ax)^6}{4320} + \dots + \frac{(-1)^n E_n (ax)^{2n}}{2n(2n)!} + \dots \\
633 \quad & \int x \operatorname{sech}^2 ax \, dx = \frac{x}{a} \operatorname{tgh} ax - \frac{1}{a^2} \ln \cosh ax \\
634 \quad & \int \frac{dx}{q + p \operatorname{sech} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q \cosh ax} \quad \text{Véase 582} \\
635 \quad & \int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \operatorname{tgh} ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx
\end{aligned}$$

#### INTEGRALES CON $\operatorname{cosech} ax$

$$\begin{aligned}
636 \quad & \int \operatorname{cosech} ax \, dx = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} \\
637 \quad & \int \operatorname{cosech}^2 ax \, dx = -\frac{\operatorname{cotgh} ax}{a} \\
638 \quad & \int \operatorname{cosech}^3 ax \, dx = -\frac{\operatorname{cosech} ax \operatorname{cotgh} ax}{2a} - \frac{1}{2a} \ln \operatorname{tgh} \frac{ax}{2} \\
639 \quad & \int \operatorname{cosech}^n ax \operatorname{cotgh} ax \, dx = -\frac{\operatorname{cosech}^n ax}{n a} \\
640 \quad & \int \frac{dx}{\operatorname{cosech} x} = \frac{\cosh ax}{a}
\end{aligned}$$

$$641 \int x \operatorname{cosech} ax \, dx = \frac{1}{a^2} \left\{ ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} - \dots + \frac{(-1)^n 2(2^{2n-1} - 1) B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}$$

$B_n$  es n° de Bernoulli

$$642 \int \frac{\operatorname{cosech} ax}{x} dx = -\frac{1}{ax} - \frac{ax}{6} + \frac{7(ax)^3}{1080} - \dots + \frac{(-1)^n 2(2^{2n-1} - 1) B_n (ax)^{2n-1}}{(2n-1)(2n)!} + \dots$$

$$643 \int x \operatorname{cosech}^2 ax \, dx = -\frac{x}{a} \operatorname{cotgh} ax + \frac{1}{a^2} \ln \sinh ax$$

$$644 \int \frac{dx}{q + p \operatorname{cosech} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q \sinh ax} \quad \text{Véase 554}$$

$$645 \int \operatorname{cosech}^n ax \, dx = -\frac{\operatorname{cosech}^{n-2} ax \operatorname{cotgh} ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosech}^{n-2} ax \, dx$$

### INTEGRALES DE FUNCIONES HIPERBOLICAS INVERSAS

$$646 \int \sinh^{-1} \frac{x}{a} dx = x \sinh^{-1} \frac{x}{a} - \sqrt{a^2 + x^2}$$

$$647 \int x \sinh^{-1} \frac{x}{a} dx = \left( \frac{x^2}{2} + \frac{a^2}{4} \right) \sinh^{-1} \frac{x}{a} - \frac{x \sqrt{a^2 + x^2}}{4}$$

$$648 \int x^2 \sinh^{-1} \frac{x}{a} dx = \frac{x^3}{3} \sinh^{-1} \frac{x}{a} + \frac{(-x^2 + 2a^2) \sqrt{a^2 + x^2}}{9}$$

$$649 \int x^n \sinh^{-1} \frac{x}{a} dx = \frac{x^{n+1}}{m+1} \sinh^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{a^2 + x^2}} dx$$

$$650 \int \frac{\sinh^{-1} \frac{x}{a}}{x} dx = \begin{cases} \frac{x}{a} - \frac{(x/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 (x/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} - \frac{1 \cdot 3 \cdot 5 (x/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots & |x| < a \\ -\frac{\ln^2(2x/a)}{2} - \frac{(a/x)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (a/x)^4}{2 \cdot 4 \cdot 4 \cdot 4} - \frac{1 \cdot 3 \cdot 5 (a/x)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} + \dots & x > a \\ -\frac{\ln^2(-2x/a)}{2} + \frac{(a/x)^2}{2 \cdot 2 \cdot 2} - \frac{1 \cdot 3 (a/x)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \frac{1 \cdot 3 \cdot 5 (a/x)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} - \dots & x < -a \end{cases}$$

$$651 \int \frac{\sinh^{-1} \frac{x}{a}}{x^2} dx = -\frac{\sinh^{-1} \frac{x}{a}}{x} - \frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 + x^2}}{x} \right)$$

$$652 \int \cosh^{-1} \frac{x}{a} dx = \begin{cases} x \cosh^{-1} \frac{x}{a} - \sqrt{x^2 - a^2}; & \cosh^{-1} \frac{x}{a} > 0 \\ x \cosh^{-1} \frac{x}{a} + \sqrt{x^2 - a^2}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$653 \int x \cosh^{-1} \frac{x}{a} dx = \begin{cases} \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \cosh^{-1} \frac{x}{a} - \frac{x \sqrt{x^2 - a^2}}{4}; & \cosh^{-1} \frac{x}{a} > 0 \\ \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \cosh^{-1} \frac{x}{a} - \frac{x \sqrt{x^2 - a^2}}{4}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$654 \int x^2 \cosh^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^3}{3} \cosh^{-1} \frac{x}{a} - \frac{x + a \sqrt{x^2 - a^2}}{9}; & \cosh^{-1} \frac{x}{a} > 0 \\ \frac{x^3}{3} \cosh^{-1} \frac{x}{a} + \frac{(x^2 + 2a^2) \sqrt{x^2 - a^2}}{9}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$655 \int x^m \cosh^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \cosh^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2 - a^2}} dx; & \cosh^{-1} \frac{x}{a} > 0 \\ \frac{x^{m+1}}{m+1} \cosh^{-1} \frac{x}{a} + \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2 - a^2}} dx; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$656 \int \frac{\cosh^{-1} \frac{x}{a}}{x} dx = \pm \left( \frac{\ln^2(2x/a)}{2} + \frac{(a/x)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (a/x)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \frac{1 \cdot 3 \cdot 5 (a/x)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} + \dots \right) \\ + \text{si } \cosh^{-1} \frac{x}{a} > 0; - \text{si } \cosh^{-1} \frac{x}{a} < 0.$$

$$657 \int \frac{\cosh^{-1} \frac{x}{a}}{x^2} dx = -\frac{\cosh^{-1}(x/a)}{x} + \frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 + x^2}}{x} \right) \\ - \text{si } \cosh^{-1} \frac{x}{a} > 0; + \text{si } \cosh^{-1} \frac{x}{a} < 0.$$

$$658 \int \operatorname{tgh}^{-1} \frac{x}{a} dx = x \operatorname{tgh}^{-1} \frac{x}{a} + \frac{a}{2} \ln(a^2 - x^2)$$

$$659 \int x \operatorname{tgh}^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 - a^2) \operatorname{tgh}^{-1} \frac{x}{a} + \frac{ax}{2}$$

$$660 \int x^2 \operatorname{tgh}^{-1} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{tgh}^{-1} \frac{x}{a} + \frac{ax^2}{6} + \frac{a^3}{6} \ln(a^2 - x^2)$$

$$661 \int x^m \operatorname{tgh}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{tgh}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{a^2 - x^2} dx$$

$$662 \int \frac{\operatorname{tgh}^{-1}(x/a)}{x} dx = \frac{x}{a} + \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} + \frac{(x/a)^7}{7^2} + \dots$$

$$663 \int \frac{\operatorname{tgh}^{-1}(x/a)}{x^2} dx = -\frac{1}{x} \operatorname{tgh}^{-1} \frac{x}{a} + \frac{1}{2a} \ln \left( \frac{x^2}{a^2 - x^2} \right)$$

$$664 \int \operatorname{cotgh}^{-1} \frac{x}{a} dx = x \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{a}{2} \ln(x^2 - a^2)$$

$$665 \int x \operatorname{cotgh}^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 - a^2) \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{ax}{2}$$

$$666 \int x^2 \operatorname{cotgh}^{-1} \frac{x}{a} dx = \frac{ax^2}{6} + \frac{x^3}{3} \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{a^3}{6} \ln(x^2 - a^2)$$

$$667 \int x^m \operatorname{cotgh}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{cotgh}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{a^2 - x^2} dx$$

$$668 \int \frac{\operatorname{cotgh}^{-1}(x/a)}{x} dx = -\left( \frac{a}{x} + \frac{(a/x)^3}{3^2} + \frac{(a/x)^5}{5^2} + \frac{(a/x)^7}{7^2} + \dots \right)$$

$$669 \int \frac{\operatorname{cotgh}^{-1}(x/a)}{x^2} dx = -\frac{1}{x} \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{1}{2a} \ln \left( \frac{x^2}{x^2 - a^2} \right)$$

$$670 \int \operatorname{sech}^{-1} \frac{x}{a} dx = \begin{cases} x \operatorname{sech}^{-1} \frac{x}{a} + a \operatorname{sen}^{-1}(x/a); & \operatorname{sech}^{-1} \frac{x}{a} > 0 \\ x \operatorname{sech}^{-1} \frac{x}{a} - a \operatorname{sen}^{-1}(x/a); & \operatorname{sech}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$671 \int x \operatorname{sech}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x}{2} \operatorname{sech}^{-1} \frac{x}{a} - \frac{a \sqrt{a^2 - x^2}}{2}; & \operatorname{sech}^{-1} \frac{x}{a} > 0 \\ \frac{x^2}{2} \operatorname{sech}^{-1} \frac{x}{a} + \frac{a \sqrt{a^2 - x^2}}{2}; & \operatorname{sech}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$672 \int x^m \operatorname{sech}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \operatorname{sech}^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{a^2 - x^2}}; & \operatorname{sech}^{-1} \frac{x}{a} > 0 \\ \frac{x^{m+1}}{m+1} \operatorname{sech}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{a^2 - x^2}}; & \operatorname{sech}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$673 \int \frac{\operatorname{sech}^{-1}(x/a)}{x} dx = \frac{1}{2} \left( \frac{\ln(x/a) \ln(4x/a)}{2} + \frac{(x/a)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (x/a)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \frac{1 \cdot 3 \cdot 5 (x/a)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} + \dots \right) \\ - \text{si } \operatorname{sech}^{-1} \frac{x}{a} > 0; + \text{si } \operatorname{sech}^{-1} \frac{x}{a} < 0.$$

$$674 \int \operatorname{cosech}^{-1} \frac{x}{a} dx = x \operatorname{cosech}^{-1} \frac{x}{a} \pm a \operatorname{senh}^{-1} \frac{x}{a}; [+ \text{si } x > 0, - \text{si } x < 0]$$

$$675 \int x \operatorname{cosech}^{-1} \frac{x}{a} dx = \frac{x^2}{2} \operatorname{cosech}^{-1} \frac{x}{a} \pm \frac{a \sqrt{x^2 + a^2}}{2}; [+ \text{si } x > 0, - \text{si } x < 0]$$

$$676 \int x^m \operatorname{cosech}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{cosech}^{-1} \frac{x}{a} \pm \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 + a^2}}; [+ \text{si } x > 0, - \text{si } x < 0]$$

$$677 \int \frac{\operatorname{cosec}^{-1}(x/a)}{x} dx = \begin{cases} \frac{\ln(x/a) \ln(4x/a)}{2} + \frac{(x/a)^2}{2 \cdot 2 \cdot 2} - \frac{1 \cdot 3 (x/a)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \dots; & 0 < x < a \\ \frac{\ln(-x/a) \ln(-x/4a)}{2} - \frac{(x/a)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (x/a)^4}{2 \cdot 4 \cdot 4 \cdot 4} - \dots; & -a < x < 0 \\ -\frac{a}{x} + \frac{(a/x)^2}{2 \cdot 3 \cdot 3} - \frac{1 \cdot 3 (a/x)^4}{2 \cdot 4 \cdot 5 \cdot 5} + \dots; & |x| > a \end{cases}$$

#### INTEGRALES DEFINIDAS

##### PROPIEDADES

$$678 \int_a^b [k f(x) + r g(x) - n h(x)] dx = k \int_a^b f(x) dx + r \int_a^b g(x) dx - n \int_a^b h(x) dx; a, b, k, r, n \in \mathbb{R}$$

$$679 \int_a^a f(x) dx = 0$$

$$680 \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$681 \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$682 \int_a^b f(x) dx = f(c) (b - a) \quad \text{Para algùn } c \text{ tal que } a < c < b, f \text{ continua en } [a; b]$$

$$683 \text{ Si } f'(x) = F(x) \Rightarrow \int_a^b f(x) dx = F(b) - F(a) \quad (\text{Regla de Barrow})$$

#### INTEGRALES IMPROPIAS

$$684 \int_a^{+\infty} f(x) dx = \lim_{z \rightarrow +\infty} \int_a^z f(x) dx$$

$$685 \int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx$$

$$686 \int_a^b f(x) dx = \lim_{\theta \rightarrow 0} \int_a^{b-\theta} f(x) dx \quad \text{Si } b \text{ es punto singular de } f, \theta > 0$$

$$687 \int_a^b f(x) dx = \lim_{\theta \rightarrow 0} \int_{a+\theta}^b f(x) dx \quad \text{Si } a \text{ es punto singular de } f, \theta > 0$$

#### INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES TRIGONOMETRICAS

$$688 \int_0^\pi \operatorname{sen} nx \operatorname{sen} kx dx = \int_0^\pi \cos nx \cos kx dx = \begin{cases} 0 & \text{si } n \neq k \\ \frac{\pi}{2} & \text{si } n = k \end{cases}; n, k \in \mathbb{Z}$$

$$689 \int_0^\pi \operatorname{sen} kx \cos nx dx = \begin{cases} 0; & \text{si } n + k \text{ es impar} \\ \frac{2k}{k^2 - n^2}; & \text{si } n + k \text{ es par} \end{cases}; n, k \in \mathbb{Z}$$

$$690 \int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x dx = \int_0^{\frac{\pi}{2}} \cos^2 x dx = \frac{\pi}{4}$$

$$691 \int_0^{\frac{\pi}{2}} \operatorname{sen}^{2k} x dx = \int_0^{\frac{\pi}{2}} \cos^{2k} x dx = \frac{1 \cdot 3 \cdot 5 \dots (2k-1)}{2 \cdot 4 \cdot 6 \dots 2k} \cdot \frac{\pi}{2} \quad k \in \mathbb{N}$$

$$692 \int_0^{\frac{\pi}{2}} \operatorname{sen}^{2k+1} x dx = \int_0^{\frac{\pi}{2}} \cos^{2k+1} x dx = \frac{2 \cdot 4 \cdot 6 \dots 2k}{1 \cdot 3 \cdot 5 \dots (2k+1)} \quad k \in \mathbb{N}$$

$$693 \int_0^{\frac{\pi}{2}} \operatorname{sen}^{2p-1} x \cos^{2q-1} x dx = \frac{\Gamma(p) \cdot \Gamma(q)}{2 \cdot \Gamma(p+q)} \quad \Gamma: \text{Función Gamma (Ver apéndice)}$$

$$694 \int_0^\infty \frac{\operatorname{sen} kx}{x} dx = \begin{cases} \pi/2 & k > 0 \\ 0 & k = 0 \\ -\pi/2 & k < 0 \end{cases}$$

$$595 \int_0^{\infty} \frac{\sin px \cos qx}{x} dx = \begin{cases} \pi/2; & 0 < p < q \\ 0; & 0 < q < p \\ \pi/4; & p = q > 0 \end{cases}$$

$$596 \int_0^{\infty} \frac{\sin px \cos qx}{x^2} dx = \begin{cases} p \pi/2; & 0 < p \leq q \\ q \pi/2; & 0 < q \leq p \end{cases}$$

$$597 \int_0^{\infty} \frac{\sin^2 px}{x^2} dx = \frac{p \pi}{2}$$

$$598 \int_0^{\infty} \frac{1 - \cos px}{x^2} dx = \frac{p \pi}{2}$$

$$599 \int_0^{\infty} \frac{\cos px - \cos qx}{x} dx = \ln \frac{q}{p}$$

$$700 \int_0^{\infty} \frac{\cos px - \cos qx}{x^2} dx = \frac{(q - p) \pi}{2}$$

$$701 \int_0^{\infty} \frac{\cos nx}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-na}$$

$$702 \int_0^{\infty} \frac{x \sin nx}{x^2 + a^2} dx = \frac{\pi}{2} e^{-na}$$

$$703 \int_0^{\infty} \frac{\sin nx}{x(x^2 + a^2)} dx = \frac{\pi}{2a^2} (1 - e^{-na})$$

$$704 \int_0^{2\pi} \frac{dx}{a + b \sin x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$$

$$705 \int_0^{\pi/2} \frac{dx}{a + b \cos x} = \frac{\cos^{-1}(b/a)}{\sqrt{a^2 - b^2}}$$

$$706 \int_0^{2\pi} \frac{dx}{(a + b \sin x)^2} = \int_0^{2\pi} \frac{dx}{(a + b \cos x)^2} = \frac{2\pi a}{\sqrt{(a^2 - b^2)^3}}$$

$$707 \int_0^{2\pi} \frac{dx}{1 - 2a \cos x + a^2} = \frac{2\pi}{1 - a^2} \quad 0 < a < 1$$

$$708 \int_0^{\pi} \frac{x \sin x dx}{1 - 2a \cos x + a^2} = \begin{cases} \frac{\pi}{2} \ln(1 + a); & \text{si } |a| < 1 \\ \pi \ln\left(1 + \frac{1}{a}\right); & |a| > 1 \end{cases}$$

$$709 \int_0^{\pi} \frac{\cos kx dx}{1 - 2a \cos x + a^2} = \frac{\pi a^k}{1 - a^2} \quad \text{si } |a| < 1, k \in \mathbb{N}$$

$$710 \int_0^{\infty} \sin ax^2 dx = \int_0^{\infty} \cos ax^2 dx = \sqrt{\frac{\pi}{8a}}$$

$$711 \int_0^{\infty} \sin ax^n dx = \frac{\Gamma(1/n)}{n a^{1/n}} \sin \frac{\pi}{2n}; \quad n > 1, \Gamma: \text{Función Gamma (Ver apéndice)}$$

$$712 \int_0^{\infty} \cos ax^n dx = \frac{\Gamma(1/n)}{n a^{1/n}} \cos \frac{\pi}{2n}; \quad n > 1$$

$$713 \int_0^{\infty} \frac{\sin x}{\sqrt{x}} dx = \int_0^{\infty} \frac{\cos x}{\sqrt{x}} dx = \sqrt{\frac{\pi}{2}}$$

$$714 \int_0^{\infty} \frac{\sin x}{x^p} dx = \frac{\pi}{2 \Gamma(p) \sin \frac{p\pi}{2}}; \quad 0 < p < 1$$

$$715 \int_0^{\infty} \frac{\cos x}{x^p} dx = \frac{\pi}{2 \Gamma(p) \cos \frac{p\pi}{2}}; \quad 0 < p < 1$$

$$716 \int_0^{\infty} \sin ax^2 \cos 2bx dx = \sqrt{\frac{\pi}{8a}} \left( \cos \frac{b^2}{a} - \sin \frac{b^2}{a} \right)$$

$$717 \int_0^{\infty} \cos ax^2 \sin 2bx dx = \sqrt{\frac{\pi}{8a}} \left( \cos \frac{b^2}{a} + \sin \frac{b^2}{a} \right)$$

$$718 \int_0^{\infty} \frac{\sin^3 x}{x^3} dx = \frac{3\pi}{8}$$

$$719 \int_0^{\infty} \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}$$

$$720 \int_0^{\infty} \frac{\operatorname{tg} x}{x} dx = \frac{\pi}{2}$$

$$721 \int_0^{\pi/2} \frac{dx}{1 + \operatorname{tg}^n x} = \frac{\pi}{4}$$

$$722 \int_0^{\pi/2} \frac{x dx}{\sin x} = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2}$$

$$723 \int_0^1 \frac{\operatorname{tg}^{-1} x}{x} dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2}$$

$$724 \int_0^1 \frac{\sin^{-1} x}{x} dx = \frac{\pi}{2} \ln 2$$

$$725 \int_0^1 \frac{1 - \cos x}{x} dx - \int_1^\infty \frac{\cos x}{x} dx = \gamma, \quad \gamma: \text{Constante de Euler}$$

$$726 \int_0^\infty \left( \frac{1}{1+x^2} - \cos x \right) \frac{dx}{x} = \gamma$$

$$727 \int_0^\infty \frac{\operatorname{tg}^{-1} px - \operatorname{tg}^{-1} qx}{x} dx = \frac{\pi}{2} \ln \frac{p}{q}$$

**INTEGRALES DEFINIDAS O IMPROPIAS  
DE FUNCIONES RACIONALES E IRRACIONALES**

$$728 \int_0^\infty \frac{dx}{x^2 + a^2} = \frac{\pi}{2a}$$

$$729 \int_0^\infty \frac{x^{p-1} dx}{1+x} = \frac{\pi}{\operatorname{sen} p\pi}; \quad 0 < p < 1$$

$$730 \int_0^\infty \frac{x^m dx}{x^n + a^n} = \frac{\pi a^{m-n+1}}{n \operatorname{sen} \left( \frac{m+1}{n} \pi \right)}; \quad 0 < m+1 < n$$

$$731 \int_0^\infty \frac{x^m dx}{1 + 2x \cos \beta + x^2} = \frac{\pi}{\operatorname{sen} m\pi} \cdot \frac{\operatorname{sen} m\beta}{\operatorname{sen} \beta}$$

$$732 \int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}$$

$$733 \int_0^a \sqrt{a^2 - x^2} dx = \frac{\pi a^2}{4}$$

$$734 \int_0^\infty x^m (a^n - x^n)^p dx = \frac{a^{m+np+1}}{n} \cdot \frac{\Gamma\left(\frac{m+1}{n}\right) \cdot \Gamma(p+1)}{\Gamma\left(\frac{m+1}{n} + p + 1\right)} \quad \Gamma: \text{Función Gamma}$$

$$735 \int_0^\infty \frac{x^m dx}{(a^n + x^n)^r} = \frac{(-i)^{r-1} \pi a^{m-nr+1}}{n \operatorname{sen} \left( \frac{m+1}{n} \pi \right) \cdot (r-1)!} \cdot \frac{\Gamma\left(\frac{m+1}{n}\right)}{\Gamma\left(\frac{m+1}{n} + r + 1\right)}; \quad 0 < m+1 < nr$$

**INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES EXPONENCIALES**

$$736 \int_0^\infty e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}$$

$$737 \int_0^\infty e^{-ax} \operatorname{sen} bx dx = \frac{b}{a^2 + b^2}$$

$$738 \int_0^\infty \frac{e^{-ax} \operatorname{sen} bx}{x} dx = \operatorname{tg}^{-1} \frac{b}{a}$$

$$739 \int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \ln \frac{b}{a}$$

$$740 \int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$741 \int_0^\infty e^{-ax^2} \cos bx dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-(b^2/4a)}$$

$$742 \int_0^\infty e^{-(ax^2+bx+c)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{(b^2-4a)/4a} \cdot f_{\operatorname{cer}} \frac{b}{2\sqrt{a}} \quad \text{Siendo } f_{\operatorname{cer}}(p) = \frac{2}{\sqrt{\pi}} \int_p^\infty e^{-k^2} dk$$

$$743 \int_0^\infty e^{-(ax^2+bx+a)} dx = \sqrt{\frac{\pi}{a}} e^{(b^2-4a)/4a}$$

$$744 \int_0^\infty x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}} \quad \Gamma: \text{Función Gamma}$$

$$745 \int_0^\infty x^m e^{-ax^2} dx = \frac{\Gamma\left(\frac{m+1}{2}\right)}{2 a^{\frac{m+1}{2}}}$$

$$746 \int_0^\infty e^{-(ax^2+b/x^2)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-2\sqrt{ab}}$$

$$747 \int_0^\infty \frac{x dx}{e^x - 1} = \sum_{n=1}^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$748 \int_0^\infty \frac{x^{n-1} dx}{e^x - 1} = \Gamma(n+1) \sum_{k=1}^\infty \frac{1}{k^n}; \quad \Gamma: \text{Función Gamma}$$

Si  $n$  es par esta serie se puede hallar con ayuda de los números de Bernoulli (ver apéndice).

$$749 \int_0^\infty \frac{x dx}{e^x + 1} = \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

$$750 \int_0^\infty \frac{x^{n-1} dx}{e^x + 1} = \Gamma(n+1) \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^n} \quad \Gamma: \text{Función Gamma}$$

$$751 \int_0^\infty \frac{\operatorname{sen} mx dx}{e^{2x} - 1} = \frac{1}{4} \operatorname{cotgh} \frac{m}{2} - \frac{1}{2m}$$

$$752 \int_0^\infty \left( \frac{1}{1+x} - e^{-x} \right) \frac{dx}{x} = \gamma \quad \gamma: \text{Constante de Euler}$$

$$753 \int_0^\infty \frac{e^{-x^2} - e^{-x}}{x} dx = \frac{\gamma}{2}$$

$$754 \int_0^{\infty} \left( \frac{1}{e^{-x}-1} - \frac{e^{-x}}{x} \right) dx = \gamma \quad \gamma: \text{Constante de Euler}$$

$$755 \int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x \sec px} dx = \frac{1}{2} \ln \frac{b^2 + p^2}{a^2 + p^2}$$

$$756 \int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x \operatorname{cosec} px} dx = \operatorname{tg}^{-1} \frac{b}{p} - \operatorname{tg}^{-1} \frac{a}{p}$$

$$757 \int_0^{\infty} \frac{e^{-ax} (1 - \cos x)}{x^2} dx = \cotg^{-1} a - \frac{a}{2} \ln(a^2 + 1)$$

#### INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES LOGARITMICAS

$$758 \int_0^1 x^m \ln^n x dx = \frac{(-1)^n n!}{(m+1)^{n+1}}; \quad m > -1, n \in \mathbb{N}_0$$

$$759 \int_0^1 x^m \ln^n x dx = \frac{(-1)^n \Gamma(n+1)}{(m+1)^{n+1}}; \quad m > -1, n \notin \mathbb{N}_0, \Gamma: \text{Función Gamma}$$

$$760 \int_0^1 \frac{\ln x}{1+x} dx = -\frac{\pi^2}{12}$$

$$761 \int_0^1 \frac{\ln x}{1-x} dx = -\frac{\pi^2}{6}$$

$$762 \int_0^1 \frac{\ln(1+x)}{x} dx = \frac{\pi^2}{12}$$

$$763 \int_0^1 \frac{\ln(1-x)}{x} dx = -\frac{\pi^2}{6}$$

$$764 \int_0^1 \ln x \ln(1+x) dx = 2(1 - \ln 2) - \frac{\pi^2}{12}$$

$$765 \int_0^1 \ln x \ln(1-x) dx = 2 - \frac{\pi^2}{6}$$

$$766 \int_0^{\infty} \frac{x^{p-1} \ln x}{1+x} dx = -\pi^2 \operatorname{cosec} p\pi \cdot \cotg p\pi \quad 0 < p < 1$$

$$767 \int_0^1 \frac{x^m - x^n}{\ln x} dx = \ln \frac{m+1}{n+1}$$

$$768 \int_0^{\infty} e^{-x} \ln x dx = -\gamma \quad \gamma: \text{Constante de Euler}$$

$$769 \int_0^{\infty} e^{-x^2} \ln x dx = -\frac{\sqrt{\pi}}{4} (\gamma + 2 \ln 2)$$

$$770 \int_0^{\infty} \ln \frac{e^x + 1}{e^x - 1} dx = \frac{\pi^2}{4}$$

$$771 \int_0^{\frac{\pi}{2}} \ln \sen x dx = \int_0^{\frac{\pi}{2}} \ln \cos x dx = -\frac{\pi \ln 2}{2}$$

$$772 \int_0^{\frac{\pi}{2}} (\ln \sen x)^2 dx = \int_0^{\frac{\pi}{2}} (\ln \cos x)^2 dx = \frac{\pi^3}{4!} + \frac{\pi \ln^2 2}{2}$$

$$773 \int_0^{\pi} x \ln \sen x dx = -\frac{\pi^2 \ln 2}{2}$$

$$774 \int_0^{\frac{\pi}{2}} \sen x \ln \sen x dx = \ln \frac{2}{e}$$

$$775 \int_0^{2\pi} \ln(a + b \sen x) dx = \int_0^{2\pi} \ln(a + b \cos x) dx = 2\pi \ln(a + \sqrt{a^2 - b^2})$$

$$776 \int_0^{\pi} \ln(a + b \cos x) dx = \pi \ln \frac{a + \sqrt{a^2 - b^2}}{2}$$

$$777 \int_0^{\pi} \ln(a^2 + 2ab \cos x + b^2) dx = \begin{cases} 2\pi \ln b; & \text{si } 0 < a \leq b \\ 2\pi \ln a; & \text{si } 0 < b \leq a \end{cases}$$

$$778 \int_0^{\frac{\pi}{4}} \ln(1 + \operatorname{tg} x) dx = \frac{\pi \ln 2}{8}$$

$$779 \int_0^{\frac{\pi}{2}} \sec x \ln \left( \frac{1 + b \cos x}{1 + a \cos x} \right) dx = \frac{(\cos^{-1} a)^2 - (\cos^{-1} b)^2}{2}$$

$$780 \int_0^a \ln \left( 2 \sen \frac{x}{2} \right) dx = - \sum_{n=1}^{\infty} \frac{\sen an}{n^2}$$

#### INTEGRALES IMPROPIAS DE FUNCIONES HIPERBOLICAS

$$781 \int_0^{\infty} \frac{\sen ax}{\sinh bx} dx = \frac{\pi}{2b} \operatorname{tgh} \frac{a}{2b}$$

$$782 \int_0^{\infty} \frac{\cos ax}{\cosh bx} dx = \frac{\pi}{2b} \operatorname{sech} \frac{a\pi}{2b}$$

$$783 \int_0^{\infty} \frac{x dx}{\sinh ax} = \frac{\pi^2}{4a^2}$$

$$784 \int_0^{\infty} \frac{x^n dx}{\sinh ax} = \frac{2^{n+1} - 1}{2^{n+1} \cdot a^{n+1}} \Gamma(n+1) \sum_{k=1}^{\infty} \frac{1}{k^{n+1}}; \quad \Gamma: \text{Función Gamma}$$

$$785 \int_0^{\infty} \frac{\sinh ax}{e^{bx} + 1} dx = \frac{\pi}{2b} \operatorname{cosec} \frac{a\pi}{b} - \frac{1}{2a}$$

$$786 \int_0^{\infty} \frac{\sinh ax}{e^{bx} - 1} dx = \frac{1}{2a} - \frac{\pi}{2b} \cotg \frac{a\pi}{b}$$

## APENDICE

### FUNCION GAMMA

Definición:  $\Gamma(n) = \int_0^{\infty} t^{n-1} e^{-t} dt; \quad n > 0$

Fórmula de recurrencia:  $\Gamma(n+1) = n \Gamma(n)$

Si  $n \in \mathbb{N} \Rightarrow \Gamma(n+1) = n!$  Si  $n < 0 \Rightarrow \Gamma(n) = \frac{\Gamma(n+1)}{n}$

Propiedades: a)  $\Gamma(p) \Gamma(1-p) = \frac{\pi}{\sin p\pi}$  b)  $\frac{2^{2x-1}}{\sqrt{\pi}} = \frac{\Gamma(2x)}{\Gamma(x) \Gamma(x + \frac{1}{2})}$

### FUNCION BETA

Definición:  $B(m, n) = \int_0^1 t^{m-1} (1-t)^{n-1} dt; \quad m > 0; n > 0$

Relación con la función Gamma:  $B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$

Otras formas de expresar la función Beta:  $B(m, n) = B(n, m) =$

$$= 2 \int_0^{\frac{\pi}{2}} \sin^{2m-1} x \cos^{2n-1} x dx = \int_0^{\infty} u^{m-1} (1+u)^{-m-n} du = r^n (r+1)^{-n} \int_0^1 \frac{t^{m-1} (1-t)^{n-1}}{(r+t)^{m+n}} dt$$

### NUMEROS DE BERNOULLI Y EULER

a) Bernoulli: los números  $B_1; B_2; B_3; \dots$  se definen por las series:

$$\frac{x}{e^x - 1} = 1 - \frac{x}{2} + \frac{B_1 x^2}{2!} + \frac{B_2 x^4}{4!} + \frac{B_3 x^6}{6!} + \dots \quad |x| < 2\pi$$

o también  $1 - \frac{x}{2} \cotg \left( \frac{x}{2} \right) = \frac{B_1 x^2}{2!} + \frac{B_2 x^4}{4!} + \frac{B_3 x^6}{6!} + \dots \quad |x| < \pi$

b) Euler: los números de Euler  $E_1; E_2; E_3; \dots$  se definen por las series:

$$\operatorname{sech} x = 1 - \frac{E_1 x^2}{2!} + \frac{E_2 x^4}{4!} - \frac{E_3 x^6}{6!} + \dots \quad |x| < \pi/2$$

$$\sec x = 1 + \frac{E_1 x^2}{2!} + \frac{E_2 x^4}{4!} + \frac{E_3 x^6}{6!} + \dots \quad |x| < \pi/2$$

Tabla de algunos números  $B_n$  y  $E_n$

$n$	$B_n$	$E_n$
1	1/6	1
2	1/30	5
3	1/42	61
4	1/30	1385
5	5/66	50521
6	691/2730	2702765
7	7/6	199360981
8	3617/510	19391512145
9	43867/798	2404879675441
10	174611/330	370371188237525

Dada  $ax^2 + bx + c = 0$

IC  $\rightarrow$  describe una curva regular  $\in D \subset \mathbb{R}^3$

$$x_1 + x_0 = -\frac{b}{a}$$

Definir  $\Gamma$  en  $c$  lo  $\text{log} \circ b \cdot c = AB$

$$x_1 \cdot x_2 = \frac{c}{a}$$

Identidades importantes.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cosh x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sinh z = \sinh x \cosh y + i \cosh x \sinh y$$

$$z \in \mathbb{C}$$

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \quad |z| < \infty$$

$$\sinh z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1} \quad |z| < \infty$$

$$\cosh z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} z^{2n} \quad |z| < \infty$$

$$\sinh z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} \quad |z| < \infty$$

$$\cosh z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} \quad |z| < \infty$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \quad |z| < 1$$

$$\frac{1}{1+z} = \sum_{n=0}^{\infty} (-1)^n z^n \quad |z| < 1$$

$$\frac{1}{1-z^2} = \sum_{n=0}^{\infty} z^{2n} \quad |z| < 1$$

$$\ln(1+z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} z^n \quad |z| < 1$$

$$(1+z)^m = 1 + \sum_{n=1}^{\infty} \frac{m(m-1)(m-2) \dots (m-n+1)}{n!} z^n \quad |z| < 1$$

$$m \in \mathbb{C}$$